Problem Set 10 Solutions

Solution to Problem 1: Well, Well, Well

Solution to Problem 1, part a.

Inside the well $V(x) = 0$ and therefore

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2}$$  \hbox{(10–6)}

Solution to Problem 1, part b.

If $E$ has a nonzero imaginary part $E_{imag}$, then the magnitude of $f(t)$ is a function of time, in particular

$$|f(t)| = \exp(E_{imag} t / \hbar)$$  \hbox{(10–7)}

If $E_{imag} > 0$ then $|f(t)|$ gets large for large values of $t$ (i.e., it blows up at infinity). If $E_{imag} < 0$ then $|f(t)|$ gets large for large values of $-t$ (i.e., it blows up at negative infinity). In either case it is impossible to normalize $\psi(x)$.

Solution to Problem 1, part c.

$$E \phi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi(x,t)}{\partial x^2}$$  \hbox{(10–8)}

Solution to Problem 1, part d.

Since

$$\phi(x) = a \sin(kx) + b \cos(kx)$$  \hbox{(10–9)}

$$\frac{d \phi(x)}{dx} = ak \cos(kx) - bk \sin(kx)$$  \hbox{(10–10)}

$$\frac{d^2 \phi(x)}{dx^2} = -ak^2 \sin(kx) - bk^2 \cos(kx) = -k^2 \phi(x)$$  \hbox{(10–11)}

Therefore

$$E \phi(x) = \left( \frac{\hbar^2 k^2}{2m} \right) \phi(x)$$  \hbox{(10–12)}

so

$$E = \frac{\hbar^2 k^2}{2m}$$  \hbox{(10–13)}
Solution to Problem 1, part e.

One of the boundary conditions is $\phi(-L) = 0$, so

$$0 = \phi(-L) = \sin(-kL) + b\cos(-kL)$$

(10–14)

The other boundary condition is $\phi(L) = 0$, so

$$0 = \phi(L) = \sin(kL) + b\cos(kL)$$

(10–15)

From this we can determine that when $a$ is nonzero, $b$ must be zero, and vice-versa.

Solution to Problem 1, part f.

$\phi(x)$ must be zero at the boundaries, which implies

$$k = \frac{j\pi}{2L}$$

(10–16)

For odd $j$, the cosine term is zero, so $a$ must be zero. For even $j$, $b$ must be zero.

Solution to Problem 1, part g.

$$e_j = \frac{\hbar^2\pi^2j^2}{8mL^2}$$

(10–17)

Solution to Problem 1, part h.

$$\phi_j(x) = \begin{cases} b\cos\left(\frac{j\pi x}{2L}\right) & j \text{ odd} \\ a\sin\left(\frac{j\pi x}{2L}\right) & j \text{ even} \end{cases}$$

(10–18)

Solution to Problem 1, part i.

$$e_1 = \frac{\hbar^2\pi^2}{8mL^2}$$

(10–19)

Solution to Problem 1, part j.

$$e_2 = \frac{\hbar^2\pi^2}{2mL^2}$$

(10–20)

Solution to Problem 1, part k.

$$e_1 = \frac{\hbar^2\pi^2}{2mL^2}$$

(10–21)

$$= \frac{(1.054 \times 10^{-34} \text{Joule-seconds})^2 \times (3.1416)^2}{8 \times (9.109 \times 10^{-31} \text{kilograms}) \times (2 \times 10^{-8} \text{meters})^2}$$

(10–22)

$$= 3.765 \times 10^{-23} \text{Joules}$$

(10–23)
Solution to Problem 1, part 1.

Express this ground-state energy in electron-volts (1 eV = 1.602 × 10^{-19} Joules).

\[
e_1 = 3.765 \times 10^{-23} \text{Joules} = 2.347 \times 10^{-4} \text{eV}
\]