Decoherence of an On-chip Oscillator

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The two devices which have the most intimate connection to the qubit are the proposed on-chip oscillator and the DC SQUID. Designing these devices requires not just concern for proper operation, but for minimum decoherence as well.

Fig 10: Circuit diagram of the SQUID oscillator coupled to the qubit. The SQUID contains two identical junctions, here represented as independent current sources and the RCSJ model, shunted by a resistor and inductor (R_{sh} and L_{sh}). A large superconducting loop (L_{mw}) provides the coupling to the qubit. The capacitor, C_c, prevents the DC current from flowing through this line, and the resistance, R_c, damps the resonance. Z_t, the impedance seen by the qubit, is the impedance across the inductor, Z_{12}.

The oscillator in Figure 10 is a simple overdamped DC SQUID. This gives two parameters with which to control the frequency and amplitude of the source: the bias current and the magnetic flux through the SQUID. In this design, the SQUID is placed on a ground plane to minimize any field bias from an external source, and direct injection supplies the flux by producing excess current along a portion of the SQUID loop. When a Josephson junction is voltage biased, its current oscillates at a frequency of \( \frac{V_{bias}}{\Phi_0} \) with an amplitude of \( I_c \). For a stable voltage bias, this looks like an independent AC current source. In this circuit, the junction is current biased, and its oscillating output produces fluctuations in the voltage at the junction. Thus the DC voltage, approximately equal to \( I_{bias} R_{sh} \), gives the fundamental frequency, while harmonics distort the signal. If the shunt is small, such that \( V_{bias} >> I_c | R_{sh} + j \omega L_{sh} | \), the voltage oscillations are small relative to the DC voltage and the higher harmonics become less of a problem. This allows us to model the junctions as independent sources (\( I_0 \) and \( I_1 \)) in parallel with the RCSJ model. A DC SQUID with a small self inductance behaves much like a single junction whose \( I_c \) can be controlled by the flux through its loop. The circuit model is shown in Figure 1. This is similar in concept to our previous work with array oscillators. The impedance seen by the qubit is given by:

\[
Z_t = \left( \frac{1}{j\omega C_c} + \frac{1}{R_c} + \frac{1}{R_{sh} + j\omega L_{sh}} \right)^{-1} + \frac{1}{j\omega L_{mw}}
\]

This value comes from placing the other elements of the circuit in parallel with the inductance. The maximum amplitude of the oscillating magnetic flux is at the resonance of the RLC circuit consisting of \( R_c, C_c \), and \( L_{mw} \). In this case, the LC resonance occurs at 8.6 GHz. Directly on resonance, the SQUID produces high amplitude oscillations with a short dephasing time. Moving it off resonance lowers the amplitude but lengthens the dephasing time, as shown in Figure 11.

<table>
<thead>
<tr>
<th>Table 1. SQUID oscillator parameters</th>
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<tr>
<td>( f )</td>
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<tr>
<td>200 MHz</td>
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<tr>
<td>100 MHz</td>
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<tr>
<td>500 MHz</td>
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The table above provides the oscillator parameters for different frequencies.
The noise from the DC SQUID shown in Figure 12 has a more complex relationship with the qubit decoherence. Since the resistive noise source is located outside of the SQUID, the noise contribution is evenly divided between the two branches. As the bias current is increased, however, the combination of circulating current and bias current creates different linear characteristics in the branches. The internal phase variable, \( \phi_{\text{int}} = (\phi_1 - \phi_2)/2 \), is driven by the external flux, so that \( \phi_{\text{int}} = \pi \Phi/\Phi_0 \). This is considered a constant. \( \phi_{\text{ext}} \) follows the bias current. While \( I_{\text{cir}} \) directly couples to the qubit, environmental noise appears as fluctuations in the \( I_{\text{bias}} \). The fluctuations in \( I_{\text{bias}} \) can be translated into fluctuations of \( I_{\text{cir}} \) through Equation (2).

By translating the noise seen on the external phase variable into noise in the circulating current, which couples to the qubit, we can derive the spectral density in Equation (3), from which we derive decoherence and dephasing times.

\[
J(\omega) = \left( \frac{2e}{\hbar} \right)^2 A_{\text{h}} \Im \{ M_{\text{h}}^* I_{\text{bias}} \phi_{\text{int}} \} \Im \{ Z_{\epsilon}(\omega) \}
\]

\( Z_{\epsilon}(\omega) \) the impedance of the external environment.

This is very similar to the above use of \( Z_{\epsilon}(\omega) \), where it was the external environment seen across the inductor, except that in this case, the external environment includes the SQUID itself (its Josephson inductance and capacitance must be included, along with any external capacitance and resistance from the environment). Notice that the decoherence caused by the SQUID increases with its bias current. Thus, when the SQUID is unbiased, it should not contribute to decoherence at all.