Electrostatically suspended torsion pendulum

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A torsion pendulum without a torsion wire has been designed and realized, in order to measure very weak forces. The arm of this torsion pendulum (5.40 g, $1.32 \times 10^{-6}$ kg m$^2$ of inertia) is electrostatically suspended. Its 6 degrees of freedom are controlled thanks to electrostatic forces, and capacitive position sensing with a noise spectral density between $10^{-10}$ and $10^{-13}$ $\text{m}/\sqrt{\text{Hz}}$. The torque noise spectral density is $1.3 \times 10^{-14}$ $\text{Nm}/\sqrt{\text{Hz}}$ around 0.05 Hz with a $1/\sqrt{f}$ increase at lower frequency, corresponding to $10^{-8}$ $\text{rad/s}^2/\sqrt{\text{Hz}}$, and $2 \times 10^{-10}$ $\text{ms}^{-2}/\sqrt{\text{Hz}}$ with a lever arm of 2 cm. The residual seismic noise limit the performances above 0.1 Hz. The free oscillating mode has a torsion stiffness of $5.14 \times 10^{-8}$ $\text{Nm/rad}$ and a $Q$ of 217. This new instrument allows on ground experiments on very weak parasitic forces inside space accelerometers developed in ONERA, with a good representativeness. For example, it is possible to measure electrostatic stiffnesses with high resolution thanks to the low torque noise spectral density; the electrostatic damping phenomenon is also well seen as illustrated by the rather low $Q$. The instrument design and operation are described, the main performances are given, and the possibilities offered are discussed. © 2000 American Institute of Physics.
plate is symmetrical to the bottom one in relation to the \( y,z \) plane. There are two pairs of electrodes for the \( y \) axis, two pairs for the \( z \) axis, and four pairs for the \( \phi \) axis. The vertical axis electrodes are at the center of the suspension plates (eight electrodes). The gap between the arm and the electrode plates is 30 \( \mu \)m wide, so that the vertical electric field can be high enough (7.7 \( \times \) \( 10^6 \) V/m) to sustain the proof mass with a reasonable voltage (228 V). This core is made of gold coated fused silica for very fine ultrasonic machining with a specific machine built at ONERA. The electrodes are surrounded by grooves, at the bottom of which the gold coating is cut, in order to have electrical separations. This prevents the dielectric parts, with their trapped charges, from being near the proof mass. The channel from the outside connections to the electrodes is also at the bottom of grooves. This limits the couplings between different signals. The core shown in Fig. 1 is inside a titanium housing (see also Fig. 2) which provides a magnetic shielding. The pressure of the residual gas is around \( 10^{-4} \) Pa thanks to an ion pump which has a magnet core that can be removed if its magnetic field perturbs the experiment.

**IV. SUSPENSION PRINCIPLE**

Each degree of freedom is sensed by a pair of electrodes, also used as actuators thanks to control voltages applied on it. The proof mass voltage is controlled through a gold wire of a few microns of diameter (see Fig. 4). This voltage is \( V_p + V_d \), where \( V_p \) is a dc voltage (nominally 10 V) and \( V_d \) is an ac voltage needed by the capacitive sensors. The gold wire is not used as a torsion wire, it is too thin to sustain the mass, and it can be mounted on the top and/or on the side of the mass.\(^7\) When mounted on the top, this wire (5 \( \mu \)m diameter and 1.8 cm long) induces a torsion stiffness of around \( 5 \times 10^{-10} \) Nm/rad, far below the total torsion stiffness of the pendulum which is in the range of \( 10^{-8} \) Nm/rad (see below).

**A. Vertical suspension**

The principle of the vertical suspension is shown in Fig. 3. The upper electrodes are used to apply control voltages (228 V in nominal conditions). The lower electrode voltages are controlled to \( V_p \), so that no force attracts the mass toward the bottom. The shape of the proof mass and of the electrodes minimizes the coupling of the vertical axes with the horizontal ones, because the proof mass is like a shield between the sets of electrodes (see also Fig. 1).

**B. Horizontal suspension**

The principle of the three horizontal axes control loops (the two translations \( y \) and \( z \) and the rotation \( \phi \)) is shown in...
The forces between the plates and the arm are proportional to the polarization voltage $V_P$ of the proof mass and to the voltage $V$ applied on the electrodes.

In order to have electrostatic negative stiffness as low as possible on the three horizontal degrees of freedom ($y$, $z$, $w$), the electrode arrangement and shapes have been chosen in order to have electrostatic forces that do not depend on the proof mass position, but only on control voltages. This is possible with a configuration where the first derivative of the capacitances with respect to the position is null. Nevertheless, some edge effects can still produce little stiffnesses.

In addition, the mass shape is circular in front of the $y$ and $z$ electrodes (see Fig. 1) so that the control forces exerted by these electrodes on the proof mass are always directed toward the mass center, and then do not produce any torque about the vertical axis: these forces do not have any lever arm. This allows us to avoid coupling of $y$ and $z$ into the $w$ degrees of freedom.

The horizontal suspension is then totally optimized for the $\varphi$ degrees of freedom.

V. ELECTRONICS

A. Capacitive sensing

The noise spectral density of the six capacitive sensors (preamplifiers and electronics) is around $10^{-7}$ pF/$\sqrt{\text{Hz}}$, with a $1/\sqrt{f}$ slope below $10^{-1}$ Hz for the vertical axes $x$, $\theta$, $\psi$, and below $10^{-2}$ Hz for the horizontal axes $y$, $z$, $\varphi$. The pitch and the yaw ($\theta$, $\psi$) are measured with a noise of around $10^{-11}$ and $2 \times 10^{-11}$ rad/$\sqrt{\text{Hz}}$. The noise along $y$ and $z$ is $3 \times 10^{-11}$ m/$\sqrt{\text{Hz}}$. The capacitive sensor along $\varphi$ has a sensitivity that can be adjusted between 150 and 95 250 V/rad. When the highest sensitivity is chosen, the angular position noise is $5 \times 10^{-10}$ rad/$\sqrt{\text{Hz}}$ (see Fig. 5) with a $1/\sqrt{f}$ increase below $10^{-2}$ Hz. With the lowest sensitivity, this noise becomes $10^{-9}$ rad/$\sqrt{\text{Hz}}$. This lowest sensitivity is chosen when large angular displacements are needed ($2.8 \times 10^{-2}$ rad are allowed between the mechanical stops).

B. Servo loops

The correcting networks are proportional, integral, and derivative (PIDs). The loops of the $X$, $\theta$, $\psi$ degrees of freedom have a bandwidth of 500 Hz. On the vertical axes, the difficulty is to use low noise capacitive sensing while having large measurement range to ensure the acquisition, and also to have a large bandwidth to compensate for the high electrostatic negative spring (128 Hz along $X$) while limiting the coupling with the high frequency detection voltage $V_d$.

The loops of the $y$, $z$, $\varphi$ axes have a bandwidth of around 0.2 Hz.

The electronics racks can be seen on the last photo of Fig. 6.

VI. PERFORMANCES

In order to be isolated from seismic noise, the torsion pendulum has been mounted on the "ONERA test bench." This apparatus has been designed to test ultrasensitive space accelerometers. The bench is a platform with horizontal axes actively decoupled from seismic noise to a level of $3 \times 10^{-9}$ m s$^{-2}$/$\sqrt{\text{Hz}}$ at 0.1 Hz with a $1/\sqrt{f}$ slope below.\(^6\) At frequencies higher than 0.1 Hz the acceleration on board the bench is due to seismic noise.

![Fig. 4. Principle of the horizontal suspension of 1° of freedom (either $y$, $z$, or $\varphi$).](image)

![Fig. 5. Capacitive sensor preamplifier and electronics noise spectrum, along $\varphi$, with a detector sensitivity of 95 250 V/ rad.](image)
The $y$ and $z$ axes of the torsion pendulum (which can be considered as a six-axes accelerometer) have measured this residual noise, showing that it is also a very good accelerometer for translation axes—see Fig. 7.

Figure 8 shows the angular acceleration spectrum measured. At frequencies higher than 0.1 Hz the electronics noise and probably the seismic noise add with a level of around $10^{-2}$ rad s$^{-2}$/Hz$^{1/2}$. It would be necessary to have a second torsion pendulum to measure the role of the seismic noise, if any. At frequencies between $10^{-4}$ and 0.1 Hz, the angular acceleration noise is $10^{-8} \sqrt{1+10^{-27}/f}$ rad s$^{-2}$/Hz. It corresponds to a torque noise of $1.3 \times 10^{-14} \sqrt{1+10^{-27}/f}$ Nm/Hz$^{1/2}$. With a lever arm of 2.2 cm, it corresponds to a force noise of $5.9 \times 10^{-13} \sqrt{1+10^{-27}/f}$ N/Hz$^{1/2}$.

It has been shown that this noise is a thermal noise coming from a damping mechanism inside the mechanics of the instrument (the fluctuation-dissipation theorem presents the link between damping and thermal noise).

The torsion pendulum $\varphi$ axis always operates with its electronics in closed loop. Nevertheless, the parameters of the free oscillating mode have been measured.

The torsion stiffness of the free oscillating mode varies with the angular position, from $5.14 \times 10^{-8}$ Nm/rad at the working point ($\varphi=0$), to $-3.0 \times 10^{-8}$ Nm/rad at $\varphi=-5 \times 10^{-8}$ rad. Figure 9 shows the torsion stiffness versus the angular position. The torsion stiffness of the gold wire mounted on the bottom of the mass has a very low contribution ($5 \times 10^{-10}$ Nm/rad), so that only electrostatic effects can produce the observed stiffness. No satisfactory theoretical estimate of the torsion constant has been possible yet, due to the difficulty of modeling the electrostatic configuration with both large and small scale elements: the proof mass has di-
dimensions of several centimeters while the gaps between the mass and the electrodes are 30 μm wide.

The $Q$ of the free oscillating mode has been computed with the formula $Q = C/(\omega_0 \times \eta(\omega_0))$, where $C$, $\omega_0$, and $\eta(\omega_0)$ are the torsion stiffness, the frequency, and the damping of the free oscillating mode. The measurement of the damping has demanded a particular effort which is described in Ref. 7.

\[
\begin{align*}
C &= 5.14 \times 10^{-8} \text{ Nm/rad} \\
J &= 1.32 \times 10^{-6} \text{ kg m}^2 \\
\omega_0 &= \sqrt{C/J} \\
n(\omega_0) &= 1.2 \times 10^{-9} \text{ N m s.}
\end{align*}
\]

This rather low $Q$ is a satisfactory consequence of the design, because it means that the damping induced by the electrostatic suspension is well seen by the instrument.

VII. DISCUSSION

The angular acceleration noise of this new instrument is quite good compared with existing torsion pendulums.\(^1\) Thanks to a light arm, the torque noise and the force noise are excellent (respectively $1.3 \times 10^{-14} \sqrt{1+10^{-2}/f} \text{ N m/Hz}$ and $5.9 \times 10^{-13} \sqrt{1+10^{-2}/f} \text{ N/Hz}$). These performances illustrate the fact that this instrument is optimized to measure very weak torques and forces, rather than accelerations.

This new instrument has been used to search for very weak parasitic forces inside ultrasensitive space accelerometers. Results are published elsewhere.\(^7\) In addition to this direct application, it may be interesting to use this torsion pendulum for fundamental physics experiments, for instance to search for new forces which could be of the order of the gravitational force at the millimeter range. This interest for fundamental physics has been raised recently\(^9\) and there is a lack of experiment at the millimeter range.\(^10\) This lack is due to the difficulty of measuring gravitational forces at low range, because it requires low masses, and forces become too weak. The present instruments have an excellent force resolution while using low masses.

To conclude, some performance improvements seem possible, since the electronics noise is around $6 \times 10^{-10} \text{ rad s}^{-2}/\sqrt{\text{Hz}}$ at 0.1 Hz. The present noise source of $10^{-8} \text{ rad s}^{-2}/\sqrt{\text{Hz}}$ at 0.1 Hz has been investigated.\(^6,7\) It appears to be a thermal noise, possibly coming from relaxation of dipolar molecules adsorbed on the proof mass. It could be reduced by cooling down the mechanical parts, and/or improving the surfaces cleanliness in order to have less dipolar molecules on it, and/or improving the design.

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\(^8\) H. Nyquist, Phys. Rev. 32, 110 (1928).