Today’s theme is going to be examples of using the Nyquist Stability Criterion. We’re going to be counting encirclements of the “-1/k” point.

But first, we must be precise about how we count encirclements.

EXAMPLE 1:

Problem: In each region, how many positive (clockwise) encirclements are there? The numbers in the regions are the answers. Notice that when counting encirclements, we are free to split the contour into any convenient number of continuous circles or loops.

CLASS EXERCISE

Label the number of clockwise encirclements in the Nyquist plots below:
Now there is only one more difficulty to resolve: what to do with singularities on the $j\omega$ axis?

\[
\frac{L(s)}{k} = \frac{1}{s}
\]

We actually have two choices:

**Choice #1**

- Radius goes to zero

**Choice #2**

- Radius goes to zero

\[
\text{Im}(\frac{L(s)}{k})
\]

\[
\text{Re}(\frac{L(s)}{k})
\]
Let’s do the accounting for the two cases. Remember that we’re calculating $Z = N + P$, and we want $Z = 0$ if the system is to be stable.

<table>
<thead>
<tr>
<th>Range of $k$</th>
<th>P</th>
<th>N</th>
<th>Z</th>
<th>Stable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\infty &lt; k &lt; 0$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>NO</td>
</tr>
<tr>
<td>$0 &lt; k &lt; \infty$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>YES</td>
</tr>
</tbody>
</table>

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<tr>
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<th>Stable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\infty &lt; k &lt; 0$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>NO</td>
</tr>
<tr>
<td>$0 &lt; k &lt; \infty$</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>YES</td>
</tr>
</tbody>
</table>

Either choice gives you the right answer! And both choices give agreement with root locus:

We have all of the basic tools now...we can proceed now to even more complex examples. There’s one more trick that can help you, which is that bode plots can be a tremendous help in drawing complicated Nyquist contours.

EXAMPLE: \[
\frac{L(s)}{k} = \frac{1}{(s+1)(10^2s+1)}
\]
Consider the Bode Plot:

\[
\frac{I}{(s+1)(10^2s+1)}
\]

\[
\log_{10} \left| \frac{L(s)}{k} \right|
\]

\[
\Delta L_0(s)
\]

\[
\text{Hits 90°@} \ \omega = 10 \Rightarrow \left| \frac{L(s)}{k} \right| = \left| \frac{I}{(10s+1)(0.1s+1)} \right| \approx 0.1
\]

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We can now do the normal accounting:

<table>
<thead>
<tr>
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<th>P</th>
<th>N</th>
<th>Z</th>
<th>Stable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\infty &lt; k &lt; -1)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>NO</td>
</tr>
<tr>
<td>(-1 &lt; k &lt; 0)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>YES</td>
</tr>
<tr>
<td>(0 &lt; k &lt; \infty)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>YES</td>
</tr>
</tbody>
</table>

And check our results with root locus:

And now an impressive example, just in case you thought EXAMPLE 1 was some kind of joke...!

\[
\frac{L(s)}{k} = \frac{(10^2s + 1)^2}{(s+1)^2(10^5s + 1)^2}
\]

Begin by doing a Bode Plot ⇒
Recitation 10: Nyquist Stability
Prof. Joel L. Dawson

The diagrams show the Bode plots for the function $\log_{10}\left|\frac{L(s)}{k}\right|$ and $\Delta L_0(s)$ versus frequency $\omega$. The plots illustrate the magnitude and phase characteristics of the system at various frequencies.
Root Locus

\[ \text{Im}\left( \frac{L(s)}{k} \right) \]

\[ \text{Re}\left( \frac{L(s)}{k} \right) \]
This is a PRIME EXAMPLE of a conditionally stable system.

Now look back at the Bode Plot, and observe where the phase crossings of $180^\circ$ are taking place. The “bubble” on the Nyquist plot where $N = 0$ is located at around $1/k = 10^{6.5}$.

Let $k=10^{6.5}$ then, and return to the Bode Plot. You’ll discover that the first time the phase hits $180^\circ$, the magnitude of the loop transmission is greater than unity!! But for this value of $k$, the Nyquist Criterion says the system is stable.

Nyquist is right...we can't trust our intuition in this case.