

## 6.302 Feedback Systems

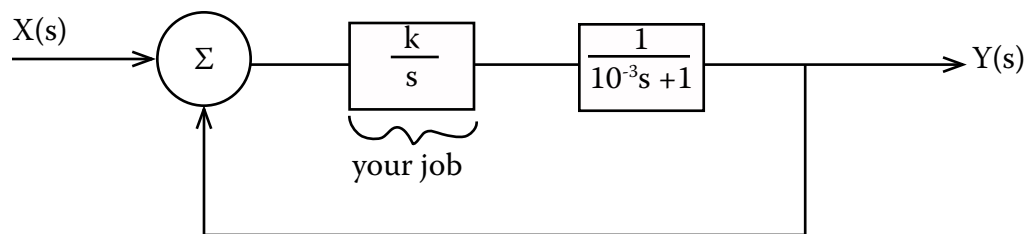
Recitation 15: More Bode Obstacle Course & Compensation

Prof. Joel L. Dawson

Today we need to wrap up some of the Bode Obstacle Course stuff that we didn't finish on Friday. Before we do, let's start with a class exercise that explores a fundamental trade-off between speed of response and stability.

### CLASS EXERCISE

Once again, you're asked to control a plant as shown:



This time, you've decided that what this system needs is a pole at the origin. Choose  $k$  to meet the following requirements:

- 1) Such that the system has a phase margin of  $\approx 90^\circ$
- 2) Such that the system has a phase margin of  $\approx 45^\circ$

For which  $k$  is the system faster?

↪ This problem illustrates a general property of feedback systems. You'll often hear people say things like, "For reasons related to stability, the bandwidth is limited to  $x$ ."

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Now, back to Bode Obstacle Course. Let's return to the example from last time:

We want to design an acceptable  $L(s)$  that results in the following closed-loop performance specs:

- 1) Steady-state error in response to a ramp  $< 1\%$
- 2) Disturbance rejection better than 10:1 for frequencies below 10 rps.
- 3) Closed-loop bandwidth  $> 50$  rps
- 4) Magnitude peaking  $M_p < 1.4$
- 5) Noise rejection better than 40 dB above  $10^3$  rps

How does this guide our decision?

- 1) Steady-state error in response to a ramp is bounded, but not zero. This implies one pole @ the origin. Let's write our loop transmission as

$$L(s) = \frac{k}{s} F(s)$$

$$\text{Where } F(s) = \frac{(\tau_{z1}s+1)(\tau_{z2}s+1)\cdots(\tau_{zN}s+1)}{(\tau_{p1}s+1)(\tau_{p2}s+1)\cdots(\tau_{pm}s+1)} \Rightarrow F(0) = 1$$

In response to a ramp, steady-state error is

$$\lim_{s \rightarrow 0} s \left( \frac{1}{s^2} \right) \frac{1}{1 + \frac{k}{s} F(s)} = \lim_{s \rightarrow 0} \frac{1}{s + kF(0)} = \frac{1}{k}$$

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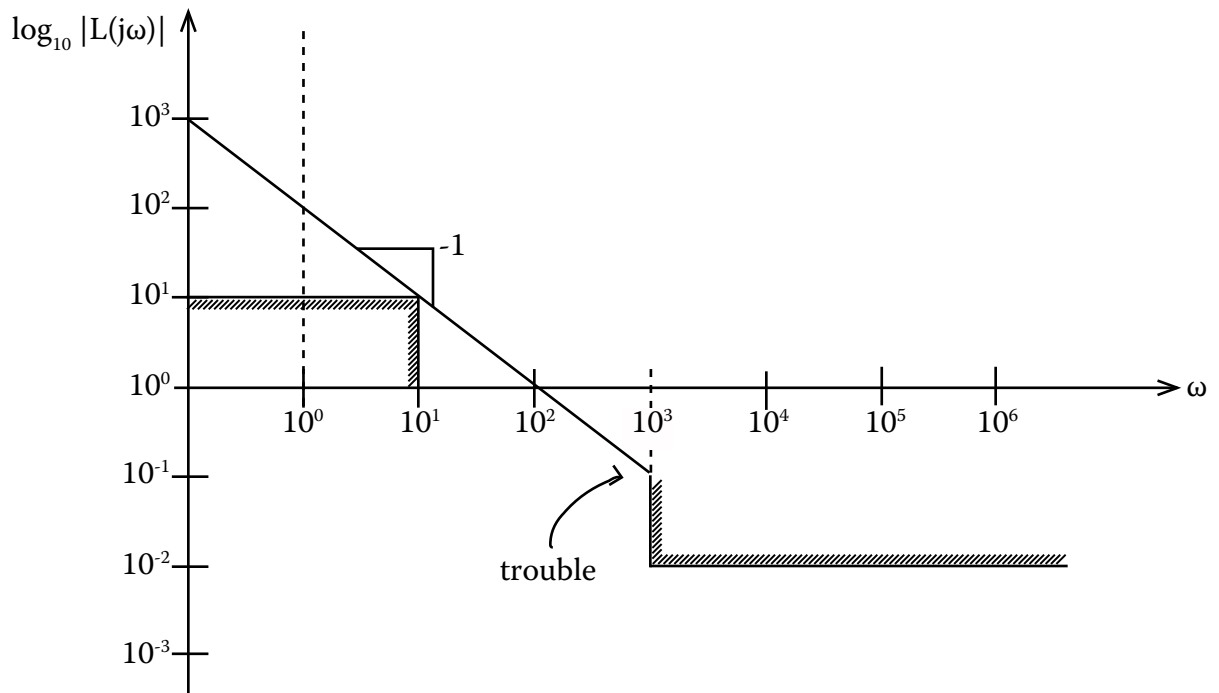
So for our first spec,

$$\frac{1}{k} < 0.01 \rightarrow k > 100$$

- 2)  $\rightarrow |L(j\omega)| > 10$  for  $\omega < 10$  rps
- 3)  $\rightarrow \omega_c > 50$  rps
- 4)  $\rightarrow \phi_m > 45^\circ$
- 5)  $\rightarrow |L(j\omega)| < 0.01$  for  $\omega > 10^3$  rps

A first try, let's follow sound engineering judgement and with the simplest  $L(s)$  possible:

$$L(s) = \frac{100}{s} .$$



What to do? We need another pole somewhere in order to meet our high-frequency spec. But if we put the pole too low in frequency, we'll lower  $\omega_c$  and our phase margin.

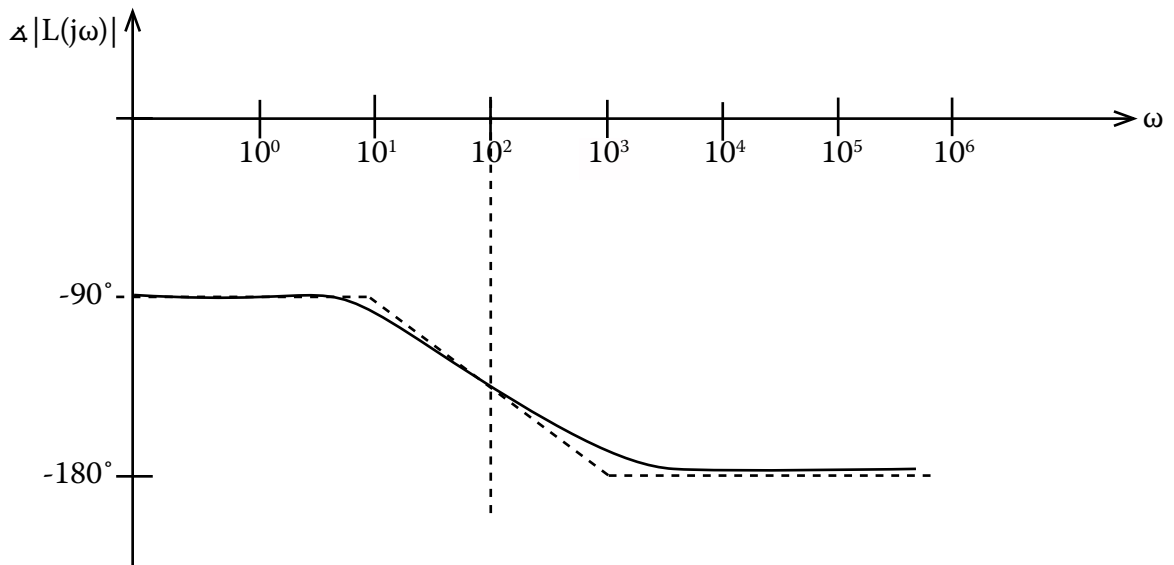
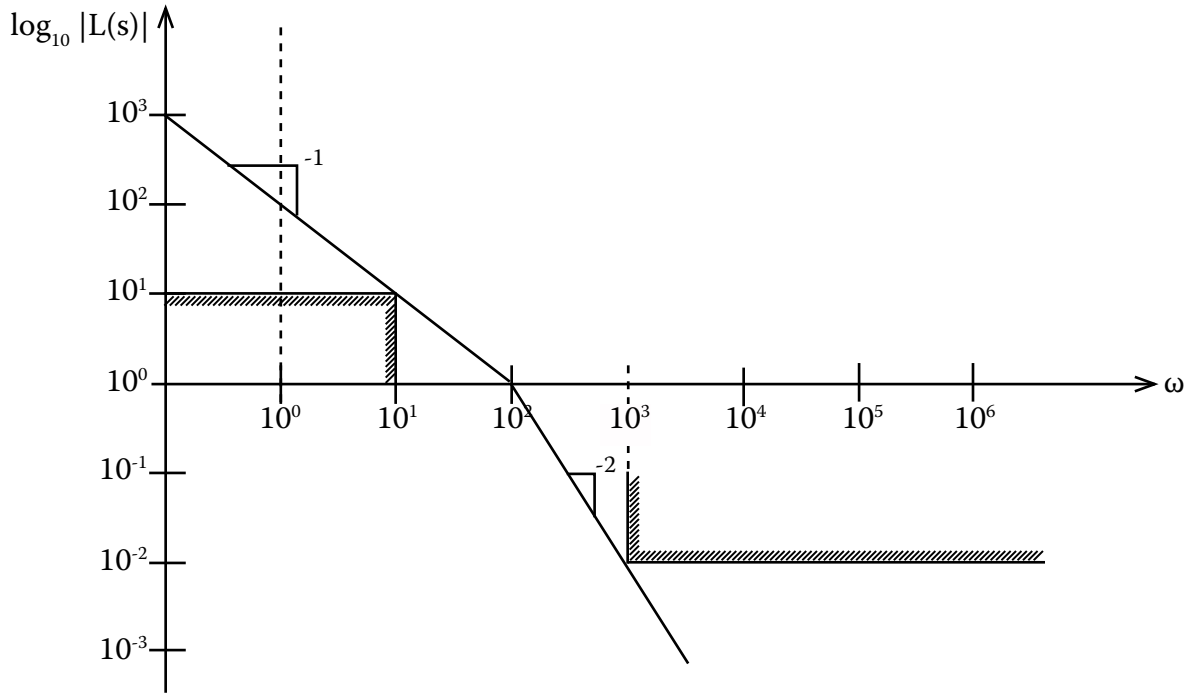
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What about a pole right at 100 rps? Using asymptotes on the bode plot, that would fix  $\omega_c$  right at 100 rps, and the phase margin would be  $45^\circ$ ....

$$\text{try } L(s) = \frac{100}{s(0.01s + 1)}$$



Actual numbers:  $\omega_c \approx 80$  rps,  $\phi_m \approx 50^\circ$ . Success!

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Remember:

- Use closed-loop specifications to place constraints on  $L(s)$
- Capture as many of those constraints as you can as Bode Obstacles.
- Start simple, and poles/zeros as necessary.

Compensation

The Bode Obstacle course is one tool we have for doing compensation, or “the art of making things better.” In our in-class exercise, we added a pole at the origin and made  $k$  as large as we could to make things better. And we noticed that there was a tradeoff between crossover frequency and stability.

So in an ideal world, what would we really want? We would want a magic box that allowed us to set its phase response independent of its magnitude response. For example, we could have arbitrary positive phase shift and a magnitude response of unity for all frequencies.

NATURE DOES NOT ALLOW THIS.

But it allows us something of that flavor. Consider a zero:

$$H(s) = \tau s + 1$$

$$\angle H(j\omega) = \tan^{-1}(\tau\omega)$$

$$|H(j\omega)| = \sqrt{1 + (\tau\omega)^2}$$

Over the range of frequencies for which  $\tau\omega \ll 1$ :

$$\left. \begin{aligned} \angle H(j\omega) &\approx \tau\omega \\ |H(j\omega)| &\approx 1 + \frac{(\tau\omega)^2}{2} \end{aligned} \right\} \begin{array}{l} \text{the phase increase is more} \\ \text{substantial than the magnitude} \\ \text{increase!} \rightarrow \text{Zeros can help.} \end{array}$$