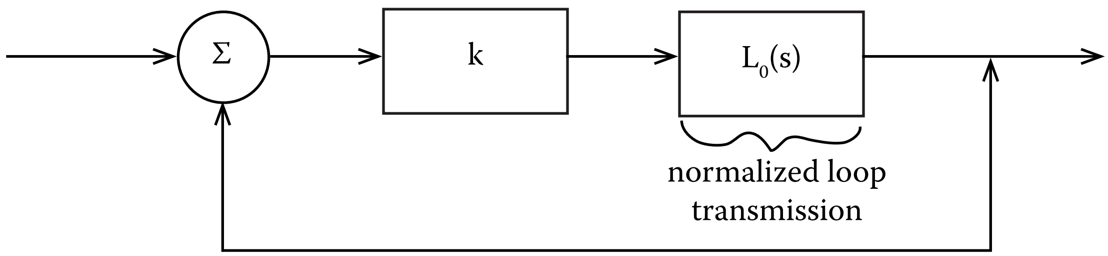
6.302 Feedback Systems

Recitation 25: Conditional Stability
Prof. Joel L. Dawson

What do we mean by "conditional" stability? The condition in question is the adjustable gain k that we assume is part of every feedback system:

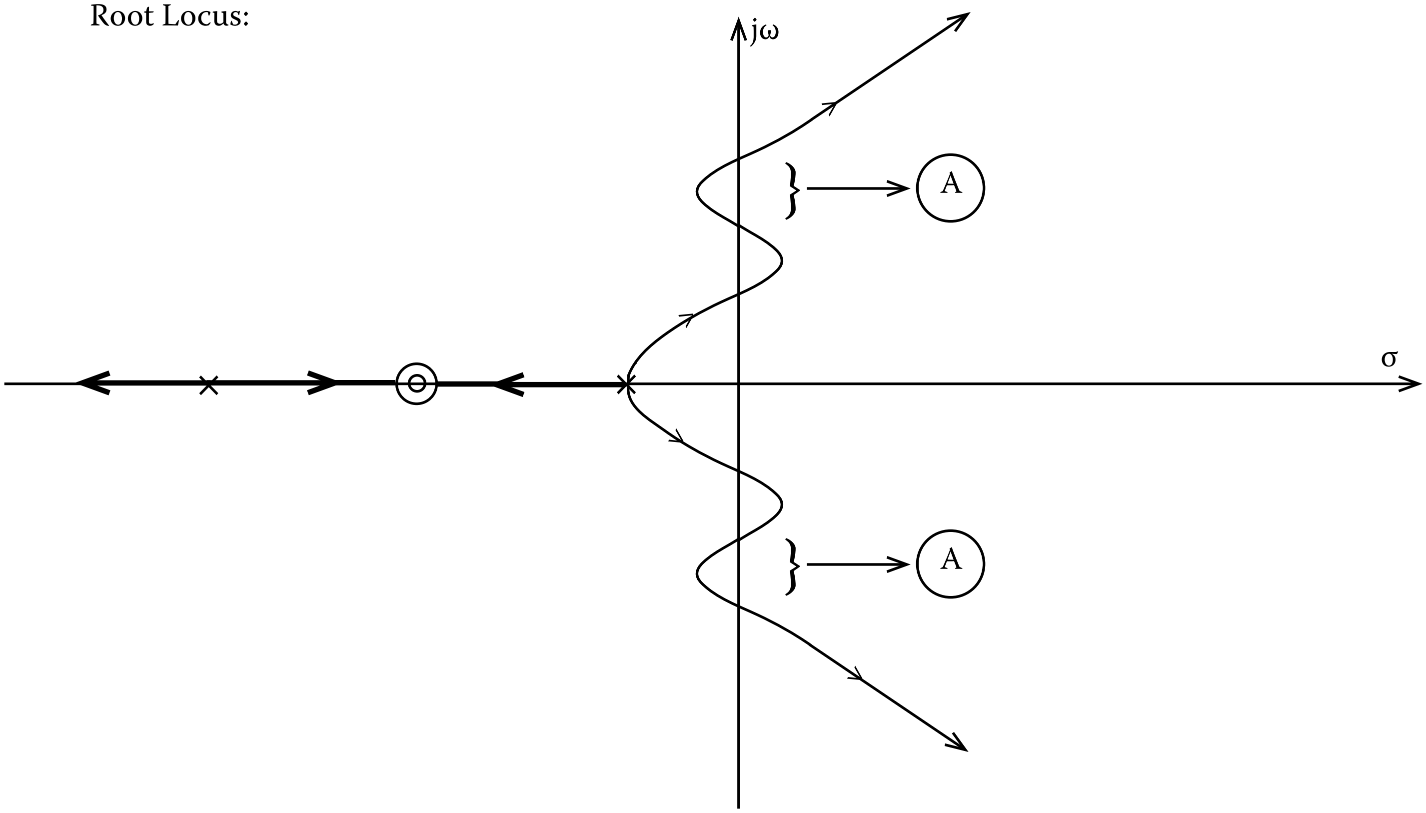


When we speak of conditional stability, we mean that the system may or may not be stable, depending on the gain k.

(Infamous) EXAMPLE

$$\frac{L(s)}{k} = \frac{(10^{-2}s + 1)^2}{(s + 1)^3\,(10^{-3}s + 1)^2}$$

Root Locus:



For gains that cause poles to be in region A, lowering the gain can actually cause instability.

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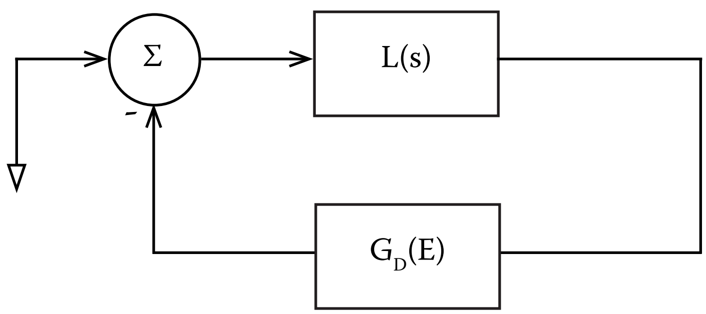
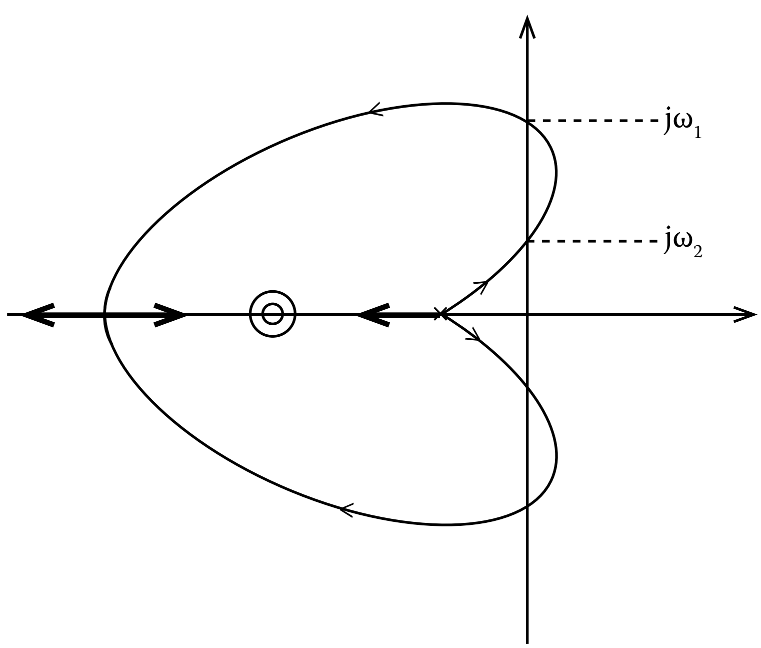
CLASS EXERCISE

1) Strictly speaking, all physical feedback systems are conditionally stable in the sense that sufficiently high loop gain will cause RHP poles. Explain this.

2) Suppose that we have a feedback system as follows:



Where $G_D(E) = \frac{4}{\pi E}$ and the root locus looks like

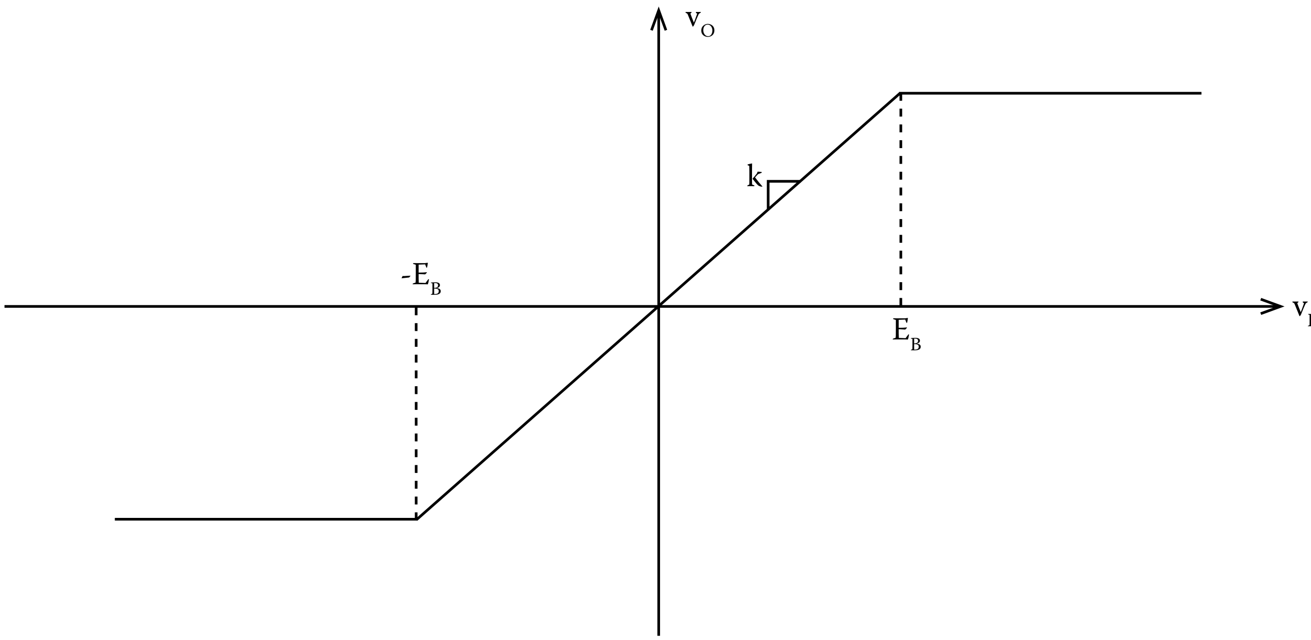


On which frequency, ω_1 or ω_2, can we have persistent oscillations?

The class exercise serves to illustrate a case where conditional stability can really be a pain: nonlinearities have a way of changing the loop gain on us.

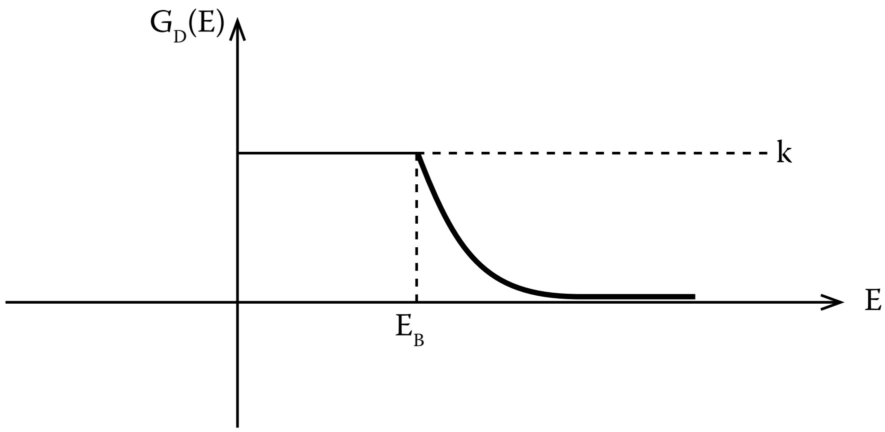
<u>Common Nonlinearities</u>

Saturation: All amplifiers exhibit this for sufficiently large inputs.

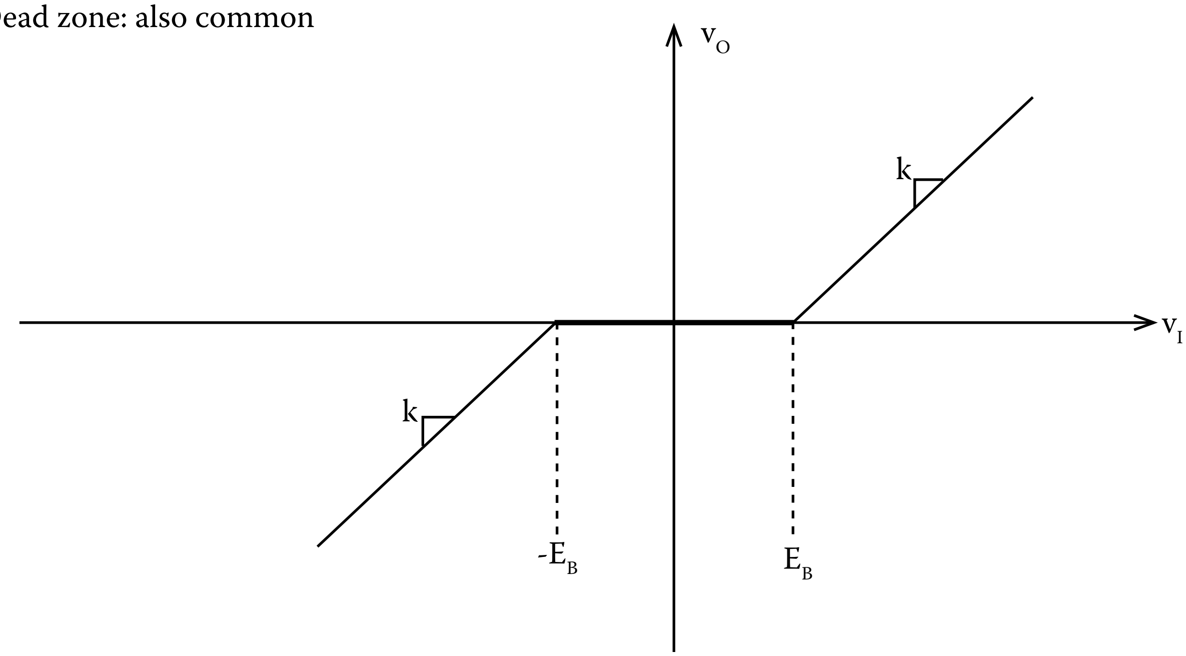


$$G_D(E) = \frac{2k}{\pi}\left[\arcsin\frac{E_B}{E} + \frac{E_B}{E}\sqrt{1-\frac{E_B^2}{E^2}}\right] \qquad \{E > E_B\}$$

Complicated! But qualitative behavior: $E \uparrow \Rightarrow G_D(E)\downarrow$



Dead zone: also common

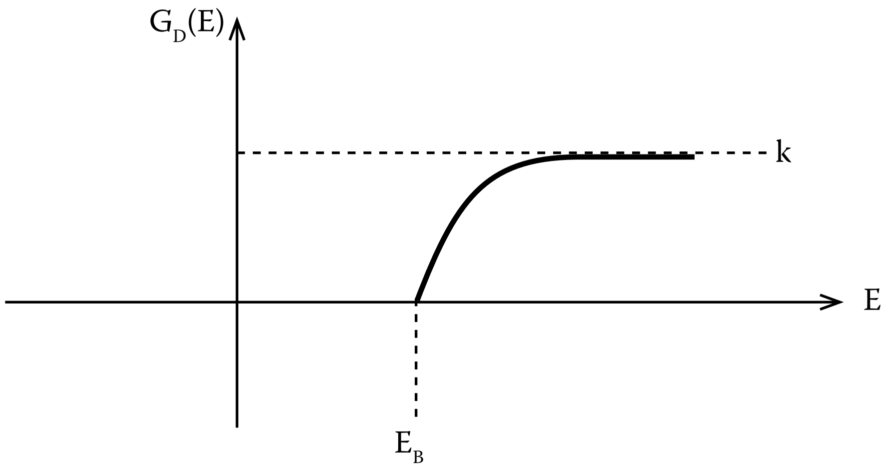


$$G_D(E) = k\left[1 - \frac{2k}{\pi}\left(\arcsin\frac{E_B}{E} + \frac{E_B}{E}\sqrt{1-\frac{E_B^2}{E^2}}\right)\right]$$

Complicated again! But qualitative behavior:

$E < 0 \Rightarrow G_D(E) = 0$

$E \uparrow \Rightarrow G_D(E) \rightarrow k$



Conditional stability is responsible for some of the "black magic" associated with analog circuit design. Ever wonder why sometimes, driving an input harder causes the system to break out into oscillations...?

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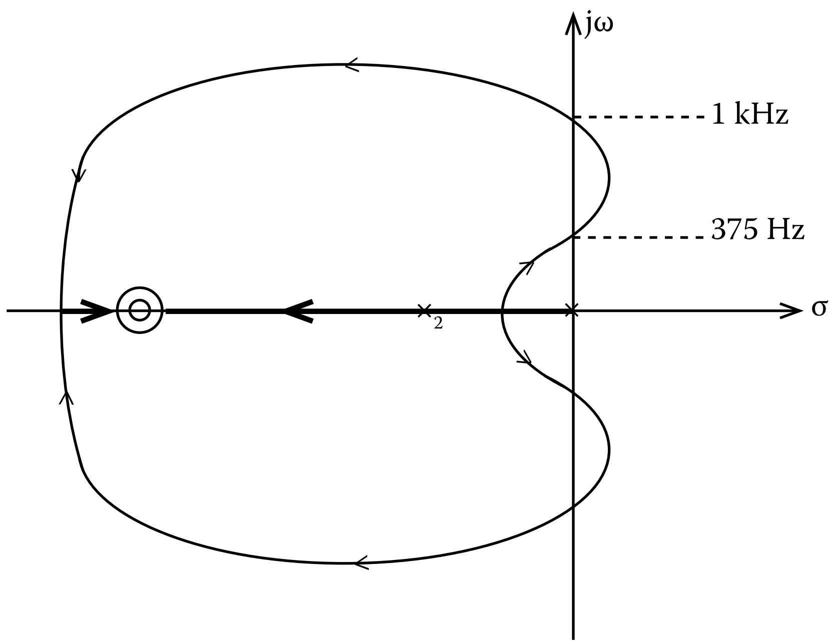
DEMONSTRATION

The demo board implements a loop transmission of

$$H(s) = \frac{k}{\tau s}\left(\frac{\tau s + 1}{\alpha\tau s + 1}\right)^2 \quad ; \tau \approx 10^{-4}\text{ sec}, \alpha \approx 7, \text{ and } 1 \leq k \leq 100.$$

Also implements hard limiting and an optional dead zone. Here's the root locus:



- Limiting nonlinearity makes persistant oscillations possible @ 375 Hz.

- Dead zone makes persistant oscillations possible @ 1 KHz