

# 6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

Quick review of Taylor Series Approximation. Suppose we have a complicated function  $f(\cdot)$ , whose value  $f(x)$  we happen to know. If the function  $f(\cdot)$  is well-behaved, we can write the equality (not merely approximation):

$$\begin{aligned} f(x + \Delta x) &= f(x) + \frac{1}{1!} \frac{df}{dx} \Delta x + \frac{1}{2!} \frac{d^2f}{dx^2} (\Delta x)^2 + \dots \\ &= f(x) + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k}{dx^k} (\Delta x)^k \end{aligned}$$

When we linearize, we approximate this equality as:

$$f(x + \Delta x) = f(x) + \Delta f \approx f(x) + \frac{df}{dx} \Delta x$$

So we identify  $\Delta f$  as  $\frac{df}{dx} \Delta x$

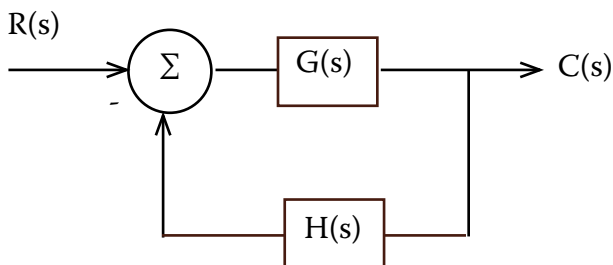
Similarly, suppose we have a function of three variables,  $f(x, y, z)$ . We still write:

$$\Delta f = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial z} \Delta z$$

We use this result over and over again when we're working with nonlinear systems.

Point of emphasis from lecture: "Trading Gain for Desensitivity"

Let's go over this slowly...



## 6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

So how sensitive is the overall transfer function to changes in  $G$ ? Using our Taylor Series Approximation, we say

$$\begin{aligned}\Delta\left(\frac{C}{R}\right) &= \frac{d}{dG} \left(\frac{G}{1+GH}\right) \cdot \Delta G \\ &= \left[G \cdot \left(-\frac{H}{(1+GH)^2}\right) + \frac{1}{1+GH}\right] \cdot \Delta G \\ &= \left[-\frac{GH}{(1+GH)^2} + \frac{1}{1+GH}\right] \cdot \Delta G \\ &= \frac{1}{(1+GH)^2} \Delta G\end{aligned}$$

More useful to us is knowing the fractional change in the transfer function given a fractional change in  $G$ . This allows us to say things like, "A 10% change in  $G$  leads to only a 0.1% change in the overall transfer characteristic." So we divide both sides of our result by  $C/R$ :

$$\frac{\Delta(C/R)}{C/R} = \frac{\frac{1}{(1+GH)^2} \Delta G}{C/R}$$

$$\text{But } C/R = \frac{G}{1+GH} \dots$$

$$\frac{\Delta(C/R)}{C/R} = \frac{\frac{1}{(1+GH)^2} \Delta G}{\frac{G}{1+GH}}$$

$$\frac{\Delta(C/R)}{C/R} = \frac{1}{1+GH} \frac{\Delta G}{G}$$

For large  $GH$  ( $GH \gg 1$ ), we get

$$\frac{\Delta(C/R)}{C/R} \approx \frac{1}{GH} \frac{\Delta G}{G}$$

# 6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

What do we now mean by “trading gain for desensitivity?” For large values of  $GH$ , the closed-loop TF is approximately  $1/H$ . So, in some sense, the only gain that we really “need” could be provided by an open-loop amplifier with gain  $1/H$ . But the forward path gain that we have is  $G$ . The amount of gain, then, that we’re “throwing away” is

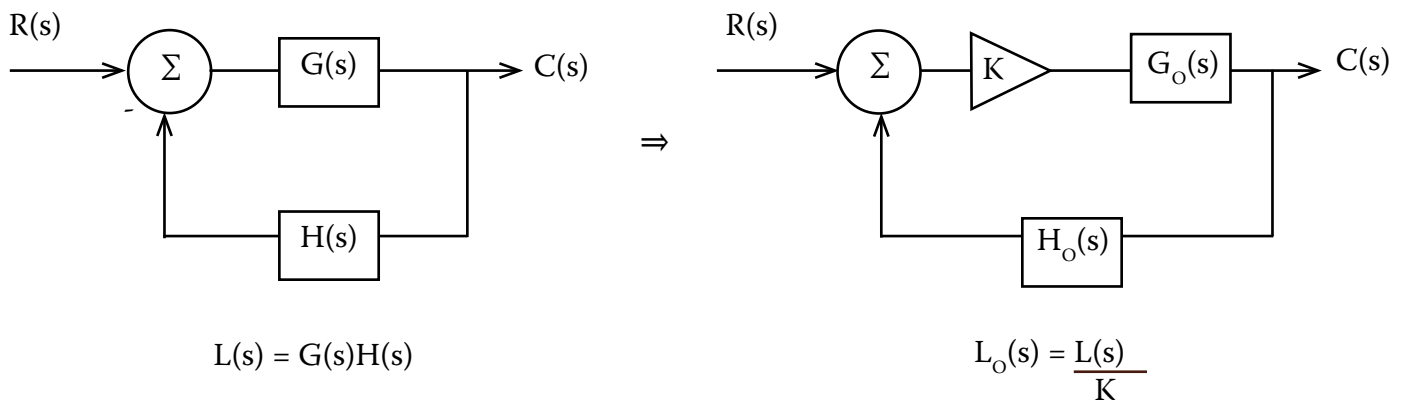
$$\frac{\text{Original forward path gain}}{\text{Gain that we need}} = \frac{G}{1/H} = GH$$

We can call  $GH$  the excess gain, or the gain that we’ve thrown away. But look: the gain that we’ve thrown away shows up again as a reduction of sensitivity to variations in  $G$ .

$$\frac{\Delta(C/R)}{C/R} \approx \frac{1}{GH} \frac{\Delta G}{G}$$

Spend some time thinking about this.

There is often an implicit assumption in our analyses that our block diagrams can be drawn in the following way:



$L_o(s)$  is the normalized loop transmission.

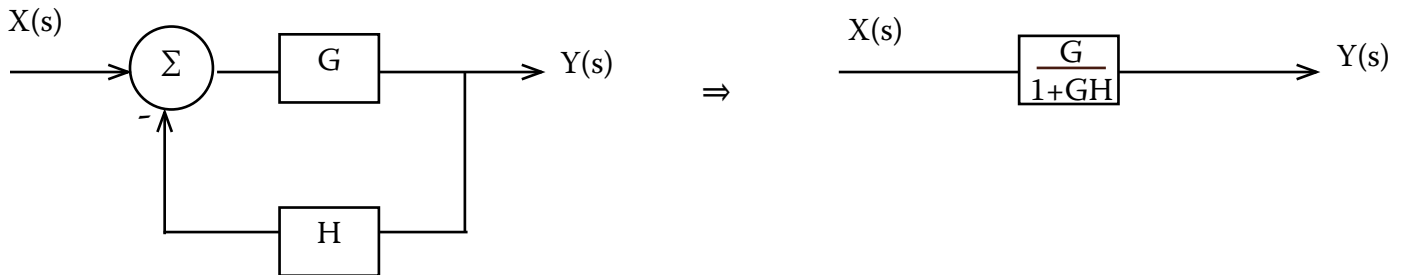
# 6.302 Feedback Systems

## Recitation 3: Block Diagrams (continued)

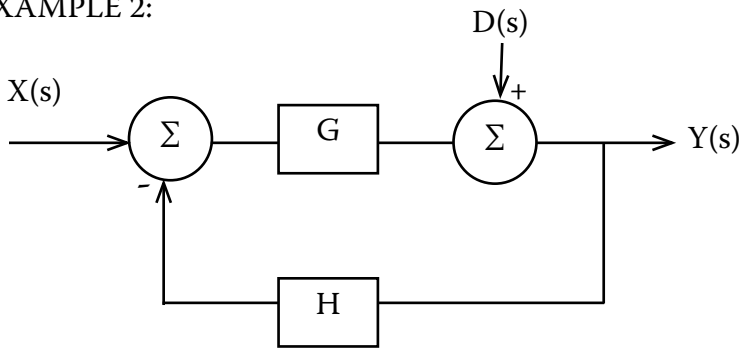
Prof. Joel L. Dawson

Now we move on to an important skill, which is manipulating block diagrams. One of the most useful tricks you will use is “collapsing” a feedback loop using Black’s Formula:

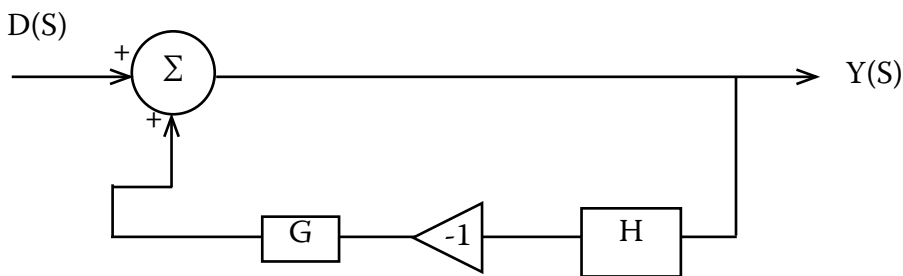
EXAMPLE 1:



EXAMPLE 2:



What is the transfer function  $Y(s)/D(s)$ ? Redraw:



Easy now!

$$\frac{Y(S)}{D(S)} = \frac{1}{1+GH}$$

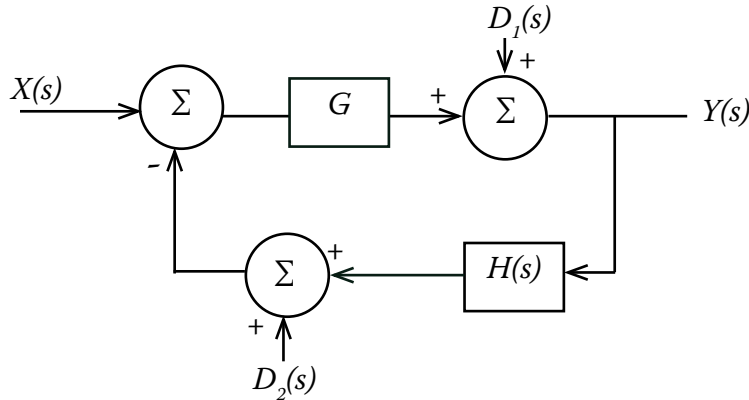
# 6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

## CLASS EXERCISE

In the following diagram, which disturbance bothers us the most? (HINT: Block diagram manipulation & Black's Formula is one, but not the only, way to reason this out.)



Other useful results: (graphical expressions of math)

① Block in series:  $X(s) \rightarrow [G_1] \rightarrow [G_2] \rightarrow Y(s) = X(s) \rightarrow [G_1 G_2] \rightarrow Y(s)$

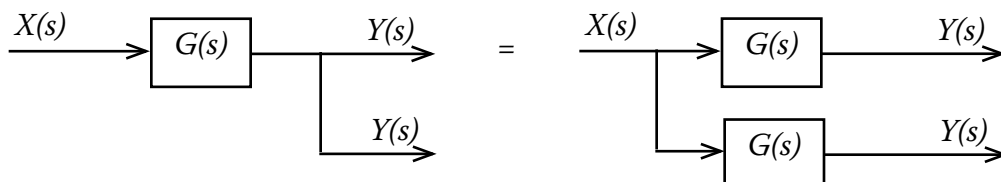
$y(t) = x(t) * g_1(t) * g_2(t) \xrightarrow{\text{LAPLACE TRANSFORM}} Y(S) = G_1(s)G_2(s)X(s)$

② Blocks in parallel:  $X(s) \rightarrow \left[ \begin{array}{c} [G] \\ [G] \end{array} \right] \rightarrow [\Sigma] \rightarrow Y(s) = X(s) \rightarrow [G_1 + G_2] \rightarrow Y(s)$

$Y(s) = G_1(s)X(s) + G_2(s)X(s)$

$= (G_1(s) + G_2(s))X(s)$

③ Blocks moved through a junction:

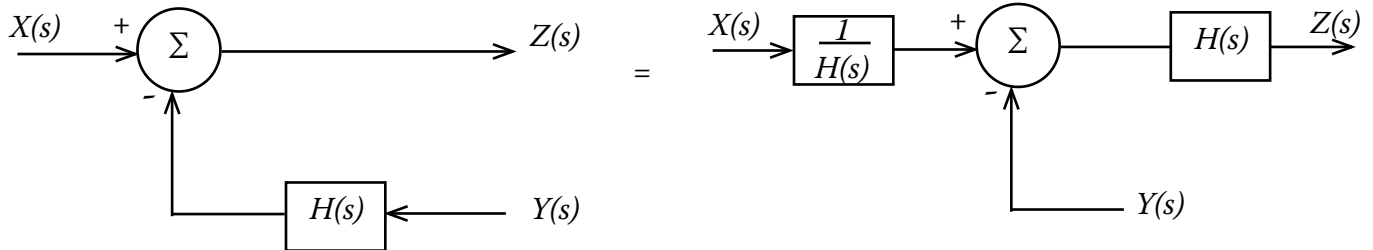


# 6.302 Feedback Systems

## Recitation 3: Block Diagrams (continued)

Prof. Joel L. Dawson

④ Moving blocks through a summer:



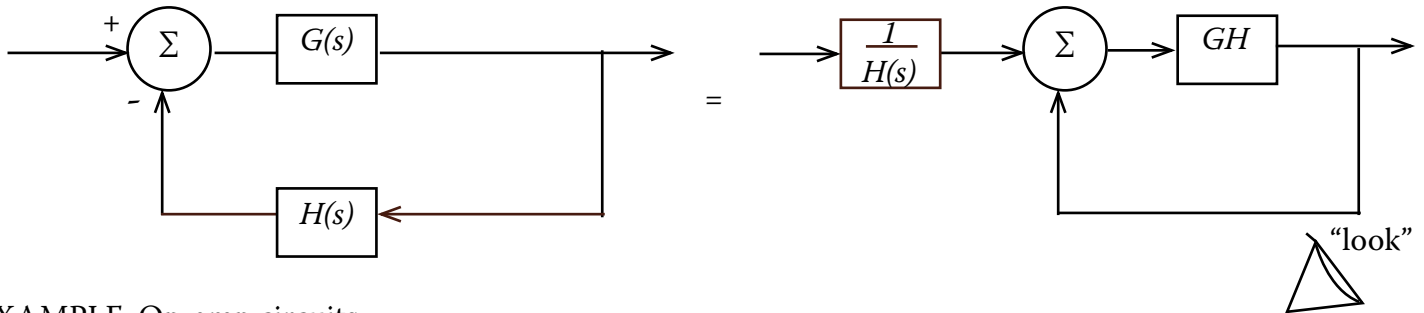
VERY USEFUL. Why does it work? Start with

$$(X - HY) = Z$$

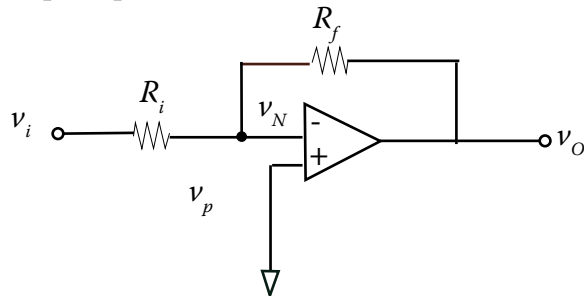
$$H \cdot 1/H (X - HY) = Z$$

$$H \left( \frac{1}{H} X - Y \right) = Z \quad \text{DONE.}$$

Useful because it lets us draw unity feedback diagrams quickly:



EXAMPLE: Op-amp circuits



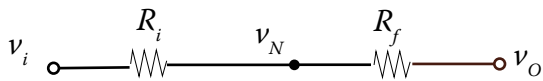
$$v_o = A(S) \cdot [v_p - v_N] = -A(S)v_N$$

must figure out  $v_N$ :  $\longrightarrow$

# 6.302 Feedback Systems

Recitation 3: Block Diagrams (continued)

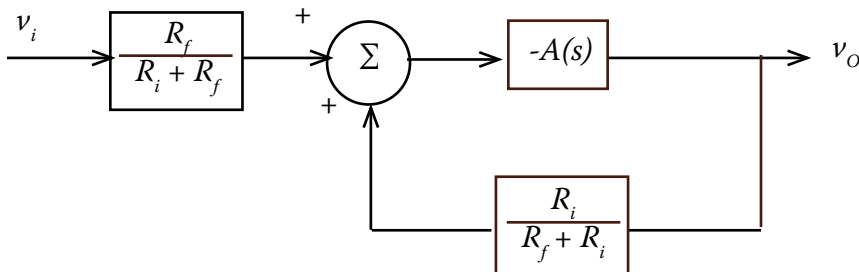
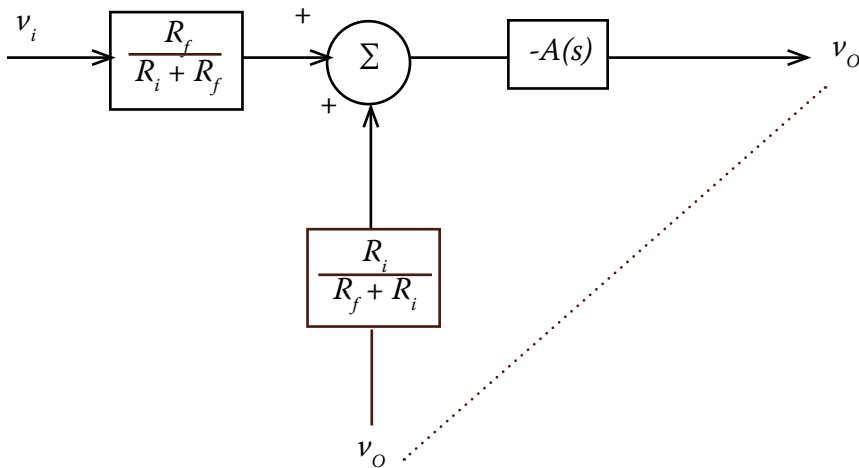
Prof. Joel L. Dawson



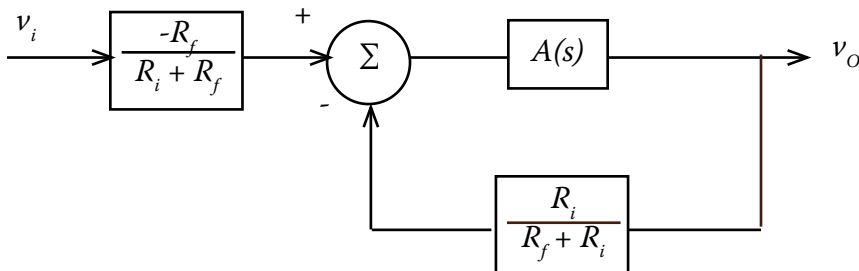
using superposition:

$$v_N = \frac{R_i}{R_i + R_f} v_o + \frac{R_f}{R_i + R_f} v_i$$

Block diagram; start with a summing junction =>



Now move minus sign through summing junction:



and draw as unity feedback: =>

