In lecture, Prof. Roberge spoke of building elaborate systems using op-amp circuits. When we do this, we habitually make statements like, “...and we’ll choose the dynamics such that the poles contributed by the op-amps themselves are negligible.” Let’s discuss.

Consider a two-pole system whose behavior interests us only in the range from DC to $\omega_0$. A pole-zero diagram:

$$\omega_{LARGE} \quad \omega_{SMALL} \quad \sigma \quad j\omega$$

The angle $\phi_L$ is small compared to $\phi_S$. => pole @ $\omega_{LARGE}$ contributes very little phase shift.

For the frequency response evaluated at $s=j\omega_0$:

$$\frac{1}{(\frac{s}{\omega_L}+1)(\frac{s}{\omega_S}+1)} \bigg|_{s=j\omega_0} = \frac{1}{(\frac{j\omega_0}{\omega_L}+1)(\frac{j\omega_0}{\omega_S}+1)}$$

$$\frac{\omega_0}{\omega_L} \ll 1 \quad \frac{\omega_0}{\omega_S} \text{ is on the order of unity (at least)!}$$

So this transfer function is well-approximated by the single-pole transfer function $\frac{1}{(\frac{s}{\omega_S}+1)}$ for frequencies from DC to $\omega_0$. 

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But what determines $\omega_0$? Depends on the context, but for feedback systems we often look at the loop transmission.

\[
\begin{align*}
\Sigma & \quad Y(s) \\
G & \quad \frac{G}{1+GH} \\
H & \quad X(s) \\
\end{align*}
\]

When $|L(s)| = |GH| \ll 1$, we arguably have an open-loop system:

\[
\frac{Y(s)}{X(s)} = \frac{G}{1+GH} \approx G
\]

So when we're looking for dynamics to ignore, we will often discard poles that are large compared to the loop crossover frequency, or the frequency at which $|L(S)| = 1$.

**Op-amp circuits for modeling our systems**

We need an integrator, a summer, an inversion, and a gain.

\[
V_o = -\frac{R_f}{R_1} V_1 + \left(\frac{R_f}{R_2}\right) V_2 + \ldots
\]
Recitation 5: Op-amp Circuits and Analog Computers
Prof. Joel L. Dawson

\[ k_i = \frac{R_f}{R_i} \]

(Note that this also covers us for an inversion.)

\[ v_o = \frac{I}{sR_iC} \]

And, a block that we do not use:

\[ \frac{v_o}{v_i} = -sCR \]

Difficult to manage in a noisy world:
Also against the differentiator: it’s hard enough to get high gain at DC. High gain at high frequencies? Forget it.

Now, on to building analog computers. Suppose we have an all-pole system. It begins as a differential equation:

\[ a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + a_{n-2} \frac{d^{n-2} y}{dt^{n-2}} + \ldots + a_0 y = x \]

Completely general procedure starts with taking the Laplace transform:

\[ (a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0)Y(s) = x(s) \]

The system function, BTW, is:

\[ \frac{Y(s)}{x(s)} = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0} \]

Solve for the highest order derivative:

\[ a_n s^n Y(s) = X(s) - (a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0)Y(s) \]

\[ s^n Y(s) = \frac{1}{a_n} X(s) - \left[ \frac{a_{n-1}}{a_n} s^{n-1} + \frac{a_{n-2}}{a_n} s^{n-2} + \ldots + \frac{a_0}{a_n} \right] Y(s) \]

Put down a big summing junction:
Generate the derivatives that you need:

\[ \sum \frac{1}{a_n} x(s) \rightarrow \frac{1}{s} s^n Y(s) \rightarrow \frac{1}{s} s^{n-1} Y(s) \rightarrow \cdots \rightarrow \frac{1}{s} Y(s) \]

Complete the mapping:

\[ \sum \frac{1}{a_n} x(s) \rightarrow \frac{1}{s} s^n Y(s) \rightarrow \frac{1}{s} s^{n-1} Y(s) \rightarrow \cdots \rightarrow \frac{1}{s} Y(s) \]

EXAMPLE: First order system

\[ \frac{Y(s)}{x(s)} = \frac{1}{\tau s + 1} \]

\[ (\tau s + 1) Y(s) = x(s) \]

\[ sY(s) = \frac{1}{\tau} x(s) - \frac{1}{\tau} Y(s) \]

=>
2nd order system from class: \[ \frac{v_o}{v_i} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} + 1} \]

With a little bit of manipulation, we can write the block diagram as

If we built this as an electronic circuit, it would be analogous to our mechanical system consisting of a mass, a viscous fluid, and a forcing mechanism:
An analog computer might look something like this:

Make sure that $\omega_n$ is small compared to the parasitic poles of the op-amps. Then, we get a very good analog.