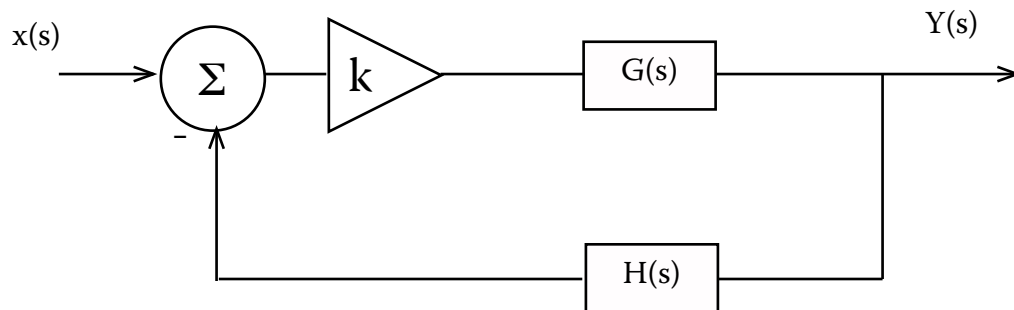


6.302 Feedback Systems

Recitation 8: Root Locus, continued

Prof. Joel L. Dawson

We've been talking about how root locus methods allow us to see how the closed-loop poles move as we vary the loop gain. But what about the closed-loop zeros??



$$\frac{Y(s)}{x(s)} = \frac{kG(s)}{1 + kG(s)H(s)} = \frac{kG(s)}{1 + kL_o(s)}$$

The closed-loop zeros are the zeros of $G(s)$ and the poles of $H(s)$. The locations of these zeros do not vary with loop gain k . Notice that we do not keep careful track of the closed-loop zero locations with root locus methods.

RULE #5: As k gets very large, $P-Z$ branches go off to infinity (rule 2). These branches approach asymptotes as angles to the real axis of

$$\alpha_n = \frac{(2n + 1) 180^\circ}{P - Z}$$

Where $n = 0 \dots (P-Z-1)$ and the centroid of these asymptotes is on the real axis at

$$\sigma_a = \frac{\sum p_i - \sum z_i}{P - Z}$$

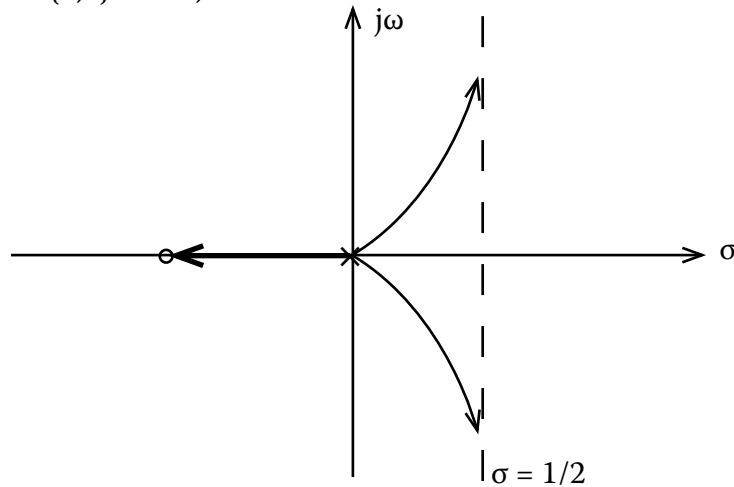
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Recitation 8: Root Locus, continued

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EXAMPLE: $L(s) = \frac{k(s+1)}{s^3}$

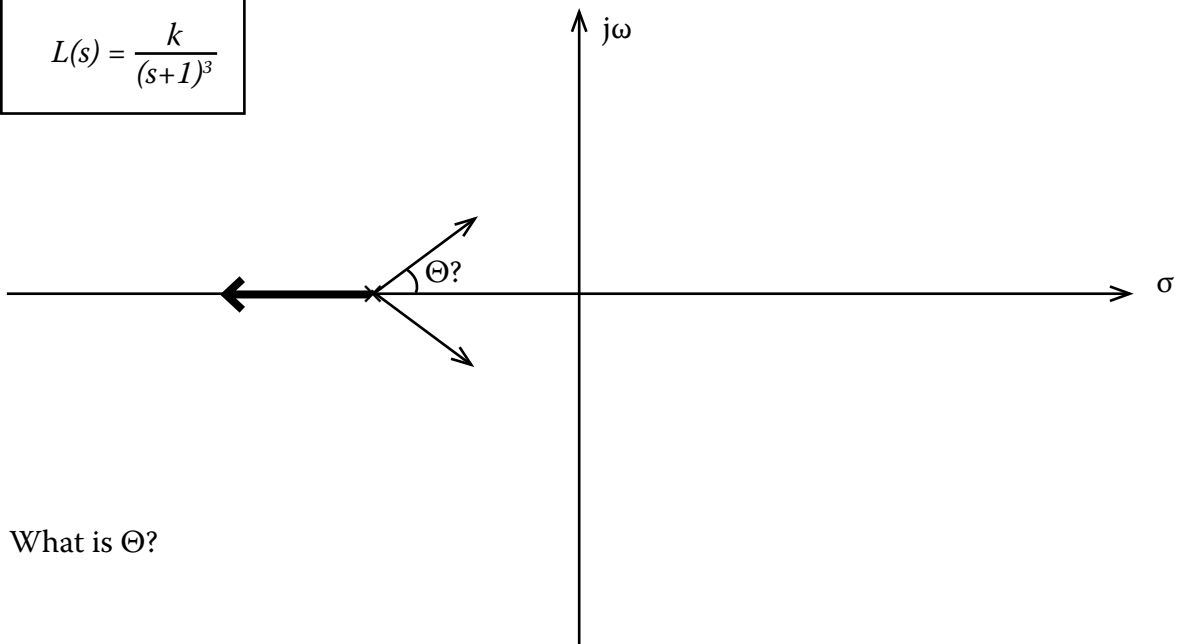
Asymptotes: $n = \{0,1\} \rightarrow 90^\circ, 270^\circ$



Centroid: $\frac{0 + 0 + 0 - (-1)}{2} = \frac{1}{2}$

RULE #6: Let's try to derive this one in class, or at least part of it. Suppose that we have a 3rd order pole sitting on the real axis. What are the angles of departure?

$$L(s) = \frac{k}{(s+1)^3}$$



What is Θ ?

6.302 Feedback Systems

Recitation 8: Root Locus, continued

Prof. Joel L. Dawson

Rule 6 is a generalization of this result. If the m th order pole is to the left of an even number of poles/zeros, the departure angles are

$$\alpha_n = \frac{(2n+1)180^\circ}{m} \quad n=\{0,1,\dots,m-1\}$$

If to the left of an odd number of singularities,

$$\alpha_n = \frac{2n \cdot 180^\circ}{m}$$

want to introduce one more rule...a complete list can be found in the text.

GRANT'S RULE: If there are two or more excess poles than zeros ($P - Z \geq 2$), then for any gain k , the sum of the real parts of the closed-loop poles is constant.

$$\begin{aligned} P(s) &= (s+a)(s+b)(s+c)(s+d) = 0 \\ P(s) &= s^4 + \omega s^3 + xs^2 + ys + z = 0 \end{aligned}$$

Coefficient ω is the sum of the roots.

$$\begin{aligned} P(s) = 1 + L(s) &= 1 + k \frac{n(s)}{d(s)} = 0 \\ d(s) + kn(s) &= 0 \end{aligned}$$

for $P-Z \geq 2$, k does not have an impact on ω .

⇒The average distance from the $j\omega$ axis remains constant,

6.302 Feedback Systems

Recitation 8: Root Locus, continued

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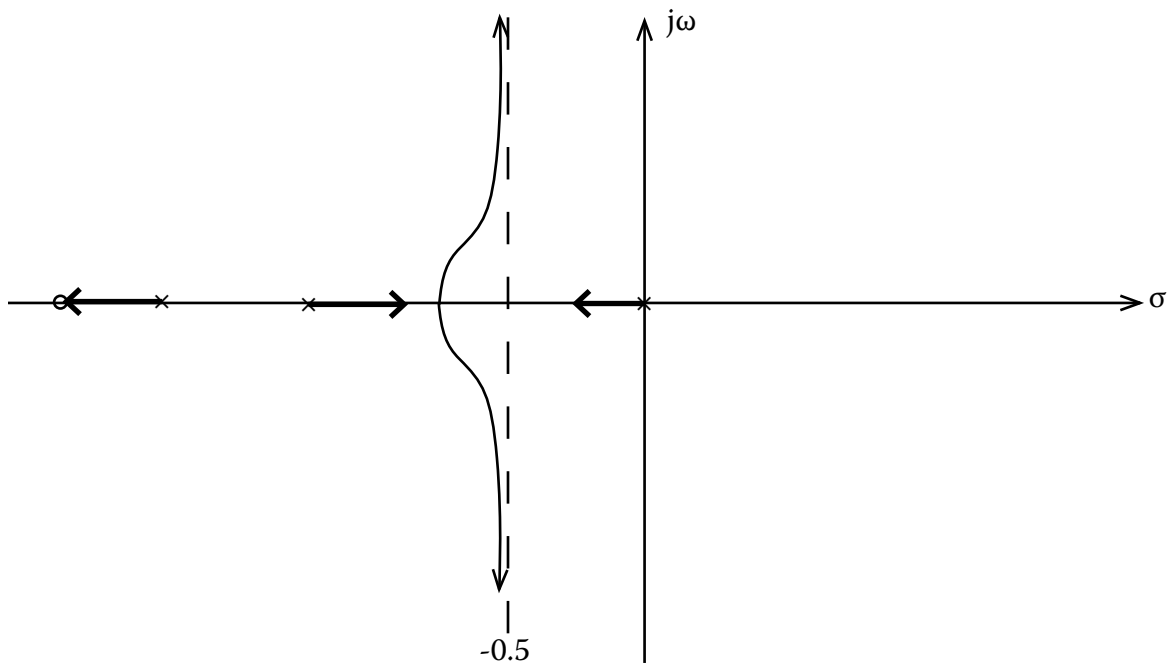
EXAMPLE:

$$L(s) = \frac{k(s+4)}{s(s+2)(s+3)}$$

Number of asymptotes: $P - Z = 2$

Angle of asymptotes: $\frac{(2m+1) \cdot 180^\circ}{2} = 90^\circ, 270^\circ$

Centroid of asymptotes: $\sigma_a = \frac{\sum p_i - \sum z_i}{P - Z} = \frac{0 - 2 - 3 + 4}{2} = -0.5$



Introduction to Compensation: Making Things Better

Root locus techniques are one of many tools that we use to design feedback systems to work the way we want them to.

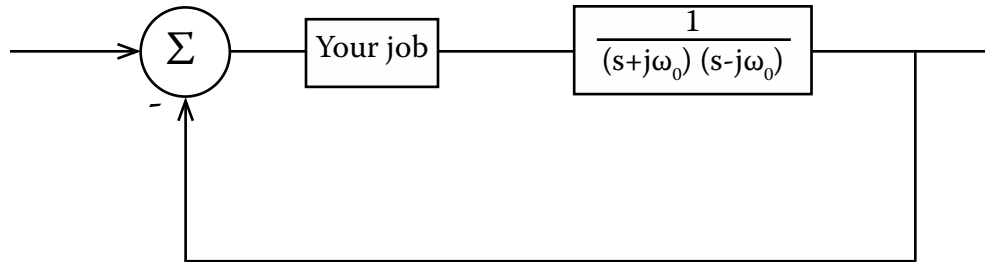
A feedback design problem might go something like this: you're given a "plant," or something you must control; the variable of interest (e.g. velocity, position, etc.), and specs on performance (maximum overshoot, rise time, bandwidth).

6.302 Feedback Systems

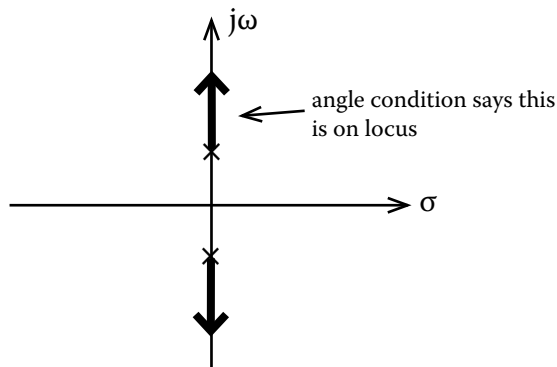
Recitation 8: Root Locus, continued

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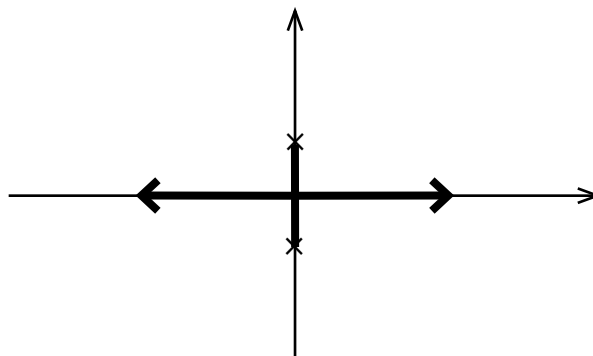
Let's say that you're asked to control a plant that has a complex pair of poles:



If you hadn't already taken 6.302, your first thought might be to put in a gain:



That didn't work: we just get an oscillator higher and higher frequency. What about negative gain (positive feedback??)? New angle condition: $\angle L_0(s) = 0$



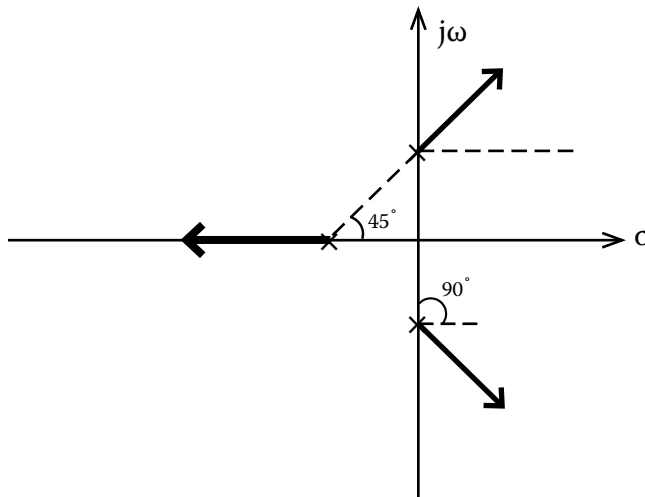
Entire real axis is on locus, as is imaginary axis between poles.

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Recitation 8: Root Locus, continued

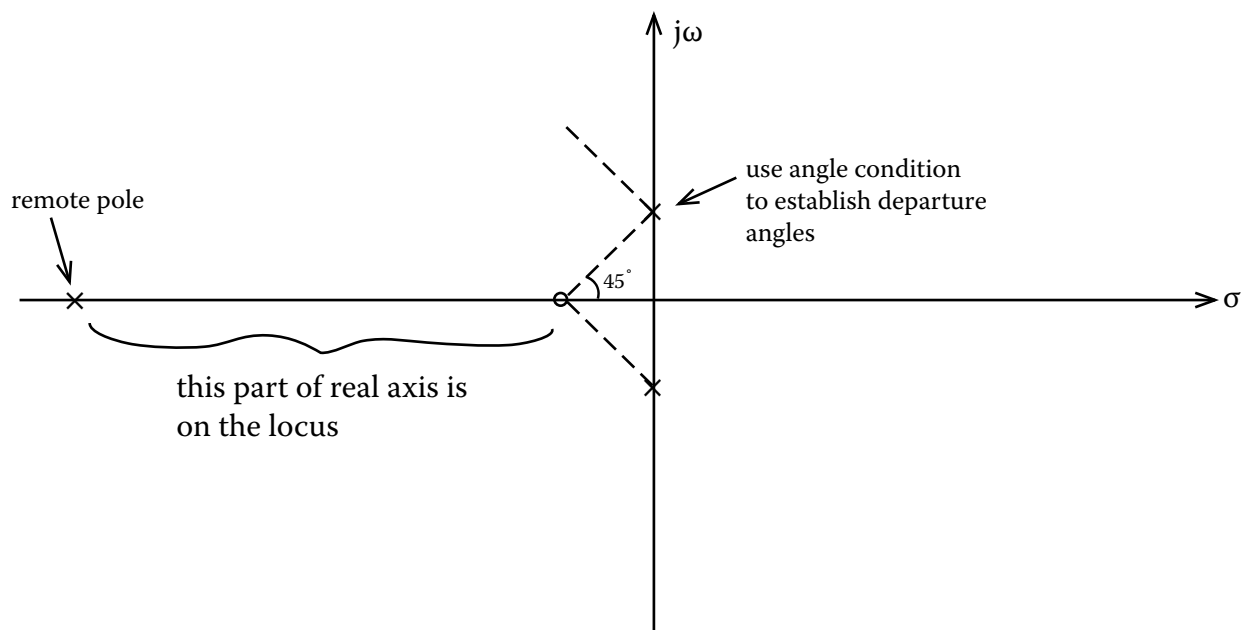
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Hmm. Try adding a real pole? (To make the numbers nice, real pole @ $-\omega_0$)



(Notice how we use angle condition to handle conjugate pair of poles).

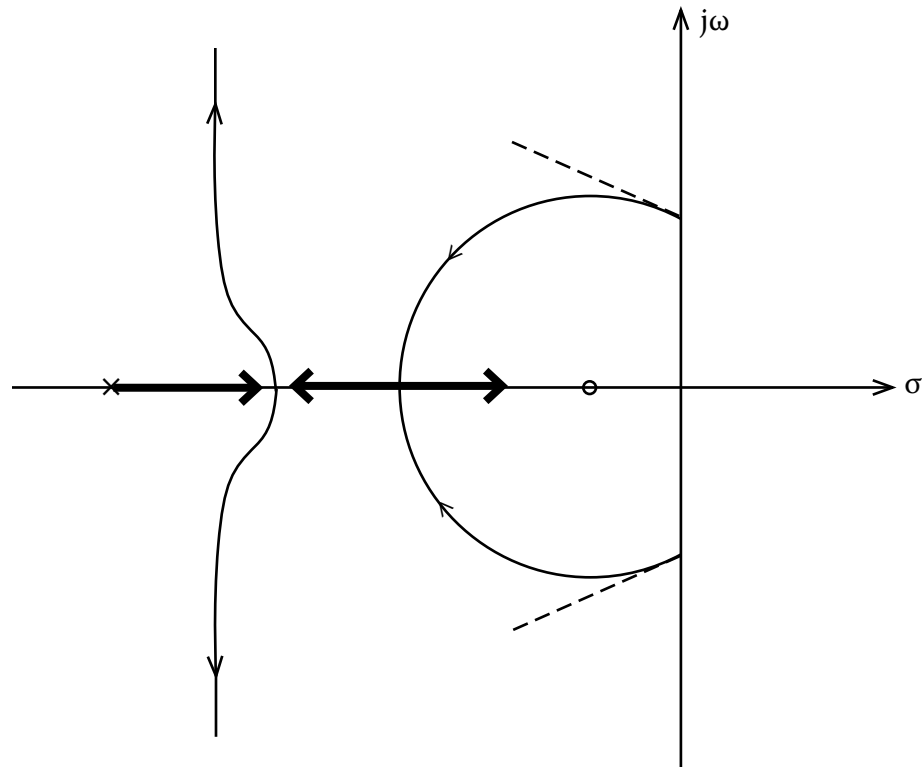
Running out of options. We can put in a zero, but we know that no physical system can have more zeros than poles. But let's try the next best thing: a zero @ $-\omega_0$ and a remote pole. If the pole is far enough away, we can ignore it in the region near the origin.



6.302 Feedback Systems

Recitation 8: Root Locus, continued

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So if we pick k right, we could wind up with only real closed-loop poles.

Notice that we haven't figured out any quantitative information (just what is k , for example?). But root locus helped us to come up with a compensation strategy.