There are times when the most important metric of an amplifier is its low-frequency gain. Based on the model we have given you so far:

We would have you believe that the achievable gain for a single stage is unbounded. That is, for a given gain, we just choose $R_L$ to be as high as necessary. As you might suspect, this is only possible up to a point.

**CLASS EXERCISE**

Consider the common emitter amplifier:

Assume that with the bias voltage $V_B$, we get perfect control over the collector current.

1. Suppose that the lowest allowable quiescent voltage at $V_0$ is zero (ground). Express the maximum $I_C$ we can have in terms of $V_{CC}$ and $R_L$.
2. What is the maximum gain we can get out of this stage? What limits us?
It is not uncommon, though, for an op-amp to achieve a gain of $10^6$ in only two stages. If we allot $10^3$ of gain for each stage, that implies a power supply $\geq 1000 \cdot V_T = 25V$. Since we routinely buy op-amps that operate on much smaller power supplies, it is clear that some other tricks are being played. One such trick looks like this:

We need a more complete model of the transistor to understand this circuit, though. Let’s jump right in…

We return to device physics. Specifically, the excess minority carrier change in the base.

The width of the base is decreased as $BC$ junction is reverse biased.
When the width of the base decreases, the slope of the carrier distribution in the base increases. This means the diffusion current must also increase. Recall,

\[ I_C = \frac{n(0)}{W_B} \times D_n \times A_E \times q \]

Now what we’re interested in is \( \frac{\partial I_C}{\partial V_{CE}} \). We don’t see an explicit dependence on \( V_{CE} \), but we know that \( W_B \) depends on \( V_{CE} \) …

\[ \frac{\partial I_C}{\partial V_{CE}} = -\frac{n_i^2}{N_B} e^{\frac{qV_{BE}}{kT}} \left( -\frac{1}{W_B} \right) D_n A_E q \frac{\partial W_B}{\partial V_{CE}} \]

Recognizing \( \frac{n_i^2}{N_B} e^{\frac{qV_{BE}}{kT}} D_n A_E q = W_B I_C \), we can simplify:

\[ \frac{\partial I_C}{\partial V_{CE}} = -\frac{1}{W_B} \frac{\partial W_B}{\partial V_{CE}} I_C \]

Now we’ve gotten as far as we can without getting into more detailed device considerations. It turns out that \( \frac{\partial W_B}{\partial V_{CE}} \) is well approximated by a constant. From a unit standpoint, we observe that

\[ \frac{1}{W_B} \frac{\partial W_B}{\partial V_{CE}} \] must have units of \( V^{-1} \). We thus write

\[ \frac{\partial I_C}{\partial V_{CE}} = kI_C = \frac{I_C}{V_A} \]
Where we call $V_A$ the “early voltage.”

What does all of this mean? For one, it means that we will modify our large signal model of the bipolar transistor in the following way:

$$I_C = I_S e^{\frac{qV_{AE}}{kT}} \left(1 + \frac{V_{CE}}{V_A}\right)$$

Second, it means that we have a new small-signal model:

$$r_0 = \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = \frac{V_A}{I_C}$$

Typical values of early voltage are 25 to 250 volts. For a collector current of $1 mA$, this gives a range between 25$k\Omega$ and 250$k\Omega$.

So how does this behavior show up in the laboratory? Most obviously, it shows up on an instrument called a curve tracer.
A curve tracer automates the graphing of $I_C$ vs $V_{CE}$ parameterized by different values of $V_{BE}$.

If you extrapolate according to the dotted lines back to where $I_C = 0$, all of the lines meet at $-V_A$.
We can see this mathematically:

$$I_C|_{V_{CE}=-V_A} = I_S e^{qV_{BE} / kT} \left(1 - \frac{V_A}{V_A}\right) = 0$$

Looking at the graph, we can see that higher early voltages imply “flatter” $I_C$ vs. $V_{CE}$ curves. And the flatter the curve, the higher $r_0$.

There’s one more effect to consider:
Notice that when we increase $V_{CE}$ and therefore decrease $W_B$, we decrease the amount of charge in the base that is available for recombination. This means that the base current decreases, and we can capture this behavior with an added resistance between the base and collector:

![Circuit Diagram]

It turns out that $r_\mu$ is typically on the order of $\beta r_0$, and usually greater than $10 \beta r_0$. We will almost always ignore this resistance.