Many times in complex analog systems, it is useful to have a current source. Consider our “active loading” concept from the last recitation:

![Diagram of transistor configuration]

Remember we speculated that if we could make the collector current of $Q_1$ match $I_{LOAD}$, this might be a way to get a very high gain stage. Infinite gain, even, were it not for base width modulation.

Now we know that transistors themselves make very good dependent current sources. This suggests their use as good independent current sources, provided that we fix the base-emitter voltage in some sensible way.

One sensible way that we looked at as follows:
This is a fine idea when you have a lot of voltage headroom. Today we’re going to look at alternatives, though, that often prove to be more useful.

CLASS EXERCISE

Compute $I_0$ for the following circuit:

![Diagram of a circuit](image)

(Workspace)

And just like that, we discover the current mirror. It’s a basic analog building block, and is one way that we have for building a good current source:
Notice how this technique solves a thorny problem for using transistors as a current source. We’ve always avoided doing things like:

Because of the uncertainty in $I_S$. But if $Q_1$ and $Q_2$ are matched, we don’t care what the exact value of $I_S$ is. We establish a reference current using the power supply and $Q_1$, and use the nonlinearity of the $I_1 - V_{BE1}$ relationship to “undo” the nonlinearity of the $V_{BE2} - I_2$ relationship. In other words, $I_{C1}$ gets mirrored into the collector of $I_{C2}$.

Remember that $I_S = A_E \cdot \left( \frac{T_n}{W_B N_B} \right)$, where $A_E$ is the emitter area. Accordingly, we can get some variety in our current mirrors by sizing the transistors differently:
This idea of using matched devices also comes in MOSFET flavors:

\[ I_0 = I_{\text{REF}} \cdot \left( \frac{W}{L} \right)_2 \]

Matching

Now we agreed that if the devices in a bipolar current mirror are matched, we have close to identical currents in the collectors.

\[ I_{c1} = I_{s1} \exp \left( \frac{qV_B}{k_T} \right) \]

\[ I_{c2} = I_{s2} \exp \left( \frac{qV_B}{k_T} \right) \]
We can conclude that for matching, we have:

\[ I_{c1} - I_{c2} = (I_{s1} - I_{s2}) \exp \left( \frac{qV_B}{kT} \right) \]

\[ \frac{\Delta I_c}{I_c} = \frac{\Delta I_s}{I_s} \]

Some common current mirrors:

**Simple Current Mirror**

\[ KCL: \ I_R = I_0 + \frac{2I_0}{\beta} = I_0 \left( 1 + \frac{2}{\beta} \right) = I_0 \left( \frac{\beta + 2}{\beta} \right) \]

\[ I_0 = I_R \left( \frac{\beta}{\beta + 2} \right) \]

We calculate the error according to \( I_0 = \frac{I_R}{1 + \frac{2}{\beta}} \)

\[ Error = \frac{2}{\beta} \]

\[ R_0 = r_0 \quad [Output \ impedance] \]
Can prevent thermal runaway, boost output impedance, and reduce sensitivity to matching by using emitter degeneration:

\[
I_B = \frac{2I_0}{(\beta+1)\beta}
\]

\[
I_R = I_0 + \frac{2I_0}{\beta(\beta+1)}
\]

\[
I_0 = \frac{I_R}{1 + \frac{2}{\beta(\beta+1)}}
\]

\[
Error = \frac{2}{\beta(\beta+1)} \approx \frac{2}{\beta^2}
\]

\[
R_0 = r_0
\]
Widlar Current Mirror (How to get very small output currents)

\[ KV_L : V_{BE1} = V_{BE2} + I_2 R_E \]

\[ V_T \ln \left( \frac{I_1}{I_S} \right) = V_T \ln \left( \frac{I_2}{I_S} \right) + I_2 R_E \]

\[ V_T \ln \left( \frac{I_1}{I_2} \right) = I_2 R_E \]

So if, for instance, we wanted \( I_1 = 1mA \) and \( I_2 = 1\mu A \):

\[ V_T \ln \left( \frac{10^{-3} \text{A}}{10^{-6} \text{A}} \right) = V_T \ln(1000) = 180mV \]

\[ I_2 R_E = 180mV \]

\[ R_E = \frac{180mV}{1\mu A} = 180k\Omega \]

The output impedance of this mirror is

\[ r_0 = r_{e2} \parallel r_E + \left(1 + g_{m2} \left( r_{e2} \parallel R_E \right) \right) r_{02} \]