Slip Flow in MicroChannels

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Abstract

Helium mass flows through microchannels (52.25 μm wide 1.33 μm deep and 7500 μm long) are measured for inlet pressures ranging from 1.2 to 2.3 atmospheres with outlet pressures at atmospheric. The effects of the slip velocity on the mass flow prediction of the Navier-Stokes equations is investigated and compared with the measured flow results. It is found that the no-slip solution of the Navier-Stokes equations fails to adequately model the momentum transferred from the fluid to the channel wall and therefore under-estimates the mass flow for given inlet and outlet pressures. However, by including a slip-flow boundary condition at the wall, which is derived from a momentum balance, we can accurately model the mass flow-pressure relationship.

1 Introduction

The characteristic length scales that govern the energy and momentum transfer between MicroElectroMechanical Systems (MEMS) and their environments are typically on the order of microns. MEMS are often operated in gaseous environments at standard conditions, where the molecular mean free path is approximately 70 nm. Hence, even at atmospheric conditions the ratio of the mean free path to the characteristic dimension can be appreciable. This ratio is known as the Knudsen number and as it increases, the exchange of energy and momentum between the systems and the environment exhibits behavior that can be attributed to the discrete molecular composition of the environment: the gas exhibits non-continuum dynamics.

Gaseous flow in micromachined channels was studied by Pfahler et al.[1] and the flow through microtubes was studied by Choi et al.[2]. Both reported a decrease in the friction factor for these types of flows which suggests a decrease in the momentum exchange between the micro system and the operating fluid. Pfahler et al.[1] identified the existence of non-continuum effects as a possible cause for this phenomena.

In this study, we develop a simple model that predicts an increase in the mass flow (decrease in friction factor) for microchannels and we present some preliminary findings.

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1.1 Slip Velocity

In solving the Navier-Stokes equation, the typical boundary condition imposed on the tangential velocity at a solid surface is that of no-slip. However, it is known that, for gaseous flow there always exists a non-zero velocity near the wall and, based on a momentum balance at the wall, it can be shown that slip velocity is a function of the velocity gradient near the wall [3], in non-dimensional form it can be shown that[4]:

\[
\frac{u_w}{c} = \frac{2 - F}{F} K_n \frac{du/c}{dy/H} \bigg|_{w},
\]

where \(u_w\) is the slip velocity at the wall, \(u\) is the \(z\) component of velocity, \(y\) is the distance normal to wall, \(K_n\) is the Knudsen number, defined as the ratio of the mean free path to characteristic dimension, \(H\) is the characteristic dimension and \(c\) is the characteristic velocity. For all gas flows, the velocity gradient is nonzero and the Knudsen number governs the order of the slip velocity. The mean free path and thus the Knudsen number is functionally dependent on the gas, and if the gas is ideal, \(K_n \propto T/P\) where \(T\) is the temperature and \(P\) is the pressure.

2 Governing Equations

The modeled channel geometry is shown in Figure 1. This is the model used for laminar, steady flow between parallel plates. The slip-velocity boundary condition at the wall on the flow profile is represented in Figure 1.

2.1 Assumptions

In order to solve the Navier-Stokes Equation, the following simplifications are made.

- Steady-flow is assumed.
The pressure drops associated with the inlet and outlet contraction/expansion are ignored.

- The flow is assumed to be isothermal.
- Cross channel flow structure in the z-direction is ignored.
- The fluid pressure is assumed to be solely a function of z, \( P = P(z) \).

3 Governing Equations

The governing equations for this study include the constant viscosity momentum equations (Navier-Stokes equations), the mass continuity equation, and the ideal equation of state. These equations, along with the slip-flow boundary condition, allow for an analysis of the microchannel flow. Non-dimensionalizing by the speed of sound, \( c \), the outlet pressure, \( P_o \), the channel length, \( L \), and the channel height, \( H \), the time-independent z-momentum equation reduces to:

\[
\frac{\rho u}{\partial z} + \frac{\rho v}{\partial y} \frac{L}{H} = -\frac{1}{\gamma} \frac{dP}{dz} + \frac{Ma_o L}{Re} \frac{\partial^2 u}{\partial y^2},
\]

(3.1)

where \( Re = \frac{m}{\rho u} \), the cross-sectionally averaged Reynolds number which is constant in \( z \), and \( Ma_o \) is the cross-sectionally averaged outlet Mach number. We have neglected \( \frac{\partial^2 u}{\partial x^2} \), it can be shown that this term is of order \( H^2/L^2 \) smaller than \( \frac{\partial^2 u}{\partial y^2} \).

The non-dimensional continuity equation can be written as:

\[
\frac{\partial(\rho u)}{\partial z} + \frac{L}{H} \frac{\partial(\rho v)}{\partial y} = 0.
\]

(3.2)

The non-dimensional equation of state is:

\[
P = \rho.
\]

(3.3)

If the convective terms can be ignored, with symmetry and the boundary condition given by Equation 1.1 we can integrate Equation 3.1 to obtain:

\[
u = -\frac{Re H}{8 \gamma Ma_o L} \frac{dP}{dz} \left( 1 - 4y^2 + 4 \frac{2 - P}{P} Kn \right),
\]

(3.8)

where \( Kn \) is the Knudsen number (based on channel height), which is functionally dependent on pressure. Normalizing by the outlet pressure, we can write \( Kn \) as a function of the outlet Knudsen number, \( Kn_o \) and the dimensionless pressure (pressure normalized by the outlet pressure). We can multiply Equation 3.8 by \( \rho \) to find the mass flow distribution in the channel, doing this yields:

\[
\rho u = -\frac{Re H}{8 \gamma Ma_o L} P \frac{dP}{dz} \left( 1 - 4y^2 + 4 \frac{2 - P}{P} \frac{Kn_o}{Kn} \right).
\]

(3.10)
In order to satisfy continuity, it can be shown that:

\[
\frac{\partial v}{\partial y} = \frac{R_e H^2}{8 M_o \gamma L T_P} \frac{\partial}{\partial z} \left[ P \frac{dP}{dz} \left( (1 - 4y^2) + 4 z F K_n \right) \right].
\]

(3.11)

If \( P \) is not a function of \( y \), Equation 3.11 can be solved for the wall normal velocity component, \( v \), which is required to satisfy differential continuity. If \( v(0) = 0 \), then:

\[
v(y) = \frac{R_e H^2}{8 M_o \gamma L T_P} \left( \frac{1}{2} \frac{d^2 P^2}{dz^2} (y - 4 y^3) + 4 z F K_n \frac{dP}{dz} y \right).
\]

(3.13)

Evaluating Equation 3.13 at the wall, where the wall normal velocity goes to zero gives:

\[
\frac{d^2 P^2}{dz^2} + 12 \frac{2 - F}{F} K_n \frac{dP}{dz} y = 0.
\]

(3.15)

This is the equation that governs the pressure distribution along the length of the channel. Solving for \( P \) gives:

\[
P(x) = -6 \frac{2 - F}{F} K_n + \frac{\left( 6 \frac{E}{F} K_n \right)^2 + \left[ P_1^2 + 12 \left( \frac{2 - E}{F} \right) K_n P_1 \right] (1 - x) + \left[ 1 + 12 \left( \frac{2 - E}{F} \right) K_n \right] x,}
\]

(3.16)

where \( P(x) \) is the normalized pressure along the channel and \( P_1 \) is the inlet to outlet pressure ratio.

With the pressure distribution we can solve Equation 3.10 for the mass flow distribution along the channel. Figure 2 is a plot of the normalized
mass flow profiles. Notice the slip mass flow present at the channel wall, $y = \pm 1/2$, increases along the channel length and in order to conserve mass the flow profiles become less steep, though the order of the reduction for the case cited is quite small and difficult to perceive in this figure. The migration of mass from the centerline, $y = 0$, to the wall which is required to preserve mass continuity is governed by Equation 3.13.

We can calculate the mass flow through the channel for given inlet and outlet pressures, by integrating Equation 3.10 across the channel in $y$ and along the channel in $x$ the *dimensional* mass flow is given by:

$$
\dot{m} = \frac{H^3 w P_o^2}{24\mu LRT} \left( P_i^2 - 1 + 12 \frac{F}{F} K_n \left( P_i - 1 \right) \right),
$$

where, $P_o$ is the outlet pressure and $P_i$ is the inlet to outlet pressure ratio.

## 4 Experimental Results

A schematic view of the channel structure is shown in Figure 3, and the channel dimensions are given in Table 1. A complete discussion of the fabrication sequence can be found in References [4] & [5].
Results of helium flow are presented below. A complete discussion of the experimental apparatus can be found in References [4] & [5]. Table 2 summarizes the experimental conditions. We can assess the validity of the initial assumptions and simplifications by noting the order of the Mach and Reynolds numbers. Due to the size of these numbers, an isothermal, laminar, non-convective flow analysis is justified.

The measured mass flow as a function of the inlet to outlet pressure ratio is shown in Figure 4 and compared to the expression given by Equation 3.20. The analytic results are plotted with the mean outlet pressure and temperature conditions. The error bars on the data at pressure ratio = 1.2, 1.85, and 2.6 represent 95% confidence intervals based on the standard deviation of numerous experiments.
4.1 Comparison with Theory

Although the outlet Knudsen number, $K_n_a$, is within the flow regime usually characterized as transitional flow, the results of the slip-model, with a specular reflection coefficient, $F = 1$, seem to fit the data nicely. The surface roughness of the prime silicon wafers was measured to be $\leq 6.5\text{Å}$ or about 3 helium molecular diameters, yet a unity specular reflection coefficient seems applicable.

5 Concluding Remarks

We have demonstrated a model that predicts an increase in the mass flow for given inlet and outlet pressures for microchannel flows that is based on the no-slip solution to the Navier-Stokes Equations. We designed and fabricated a micromachined device that allowed us to verify this model. It has been shown that for large Knudsen number flows, the flow in microchannels may not be modeled with the no-slip boundary condition.

Currently we are implementing microchannels that will allow for measurement of the pressure distribution along the length of the channel and are continuing flow analysis with various gaseous species.

References


